

Analyzing competition over visibility in social networks

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Abstract: Social Networks have known an important evolution in the last few years. These structures, made up of individuals who are tied by one or more specific types of interdependency, constitute the window for members to express their opinions and thoughts by sending posts to their own walls or others' timelines. Actually, when a content arrives, it's located on the top of the timeline pushing away older messages. This situation causes a permanent competition over visibility among subscribers who jump on opponents to promote conflict. Our study presents this competition as a non-cooperative game; each source has to choose frequencies which assure its visibility. We model it, exploring the theory of concave games, to reach a situation of equilibrium; a situation where no player has the ultimate ability to deviate from its current strategy. We formulate the named game, then we analyze it and prove that there is exactly one Nash equilibrium which is the convergence of all players' best responses. We finally provide some numerical results, taking into consideration a system of two sources with a specific frequency space, and analyze the effect of different parameters on sources' visibility on the walls of social networks.

Keywords: Social networks, game theory, Nash Equilibrium, best response, utility function, concave game.

1. Introduction

The rise of the internet came up with a digital technique permitting to construct social and active environments of interactions and relationships. Those environments connect social actors called nodes, with each other by certain types of interdependency such as: friendship, common interest, financial exchange, dislikes, relationships of beliefs and knowledge.

Moreover, these structures, which are considered as sets of participants (individuals, organizations...), are changing the way of interacting via Internet. To be one of the users and interact with others, a source has to share some personal information but also to post messages on timelines. Therefore, timelines will not allow the visibility of all the arrival posts because of their limited capacity and visibility limitation. To overcome these limitations, selfish sources have to post more contents in order to keep themselves visible on a specific wall. Henceforth, this selfishness leads to a competition over visibility.

To analyze this competition and model it a variety of techniques took place, including those from network science and game theory. Therefore, the game theoretic approach can offer new views beyond what classical methods could do. This mathematical discipline interested in strategic decision-making, modeling and analyzing interactions among individuals is launched on 1944 by John Von Neumann (mathematician) and

Morgenstern (economist) who justified the idea to maximize the expected payoff in their work in [1] and followed by contributions of Nash (1950) [2].

This theory is a basic tool which is used in many disciplines [3] among whom we can name: economy [1], Biology [4, 5] and road traffic [6], It is also applied in telecommunication networks [7, 8] to analyze situations and find a point of equilibrium (Price and Quality of Service) among SPs.

Hence, the purpose of the theory is to balance the interactions between players, to predict and define the possible outcomes of such interactions, and to develop strategies of players which lead to an optimal outcome of the game. One of the principle keys of the game theory is the dependence of the players' actions on the choices of other players, more than their own choices.

The main topic of this work is to perform behavior modeling in social networks using the theory of concave games which is introduced by J. Rosen in [9] assuring an interesting analysis for the existence and the uniqueness of the equilibrium point of a game. Following [9] we aim to develop an efficient approach which gives a situation of equilibrium solving the competition over visibility on social networks' timelines.

To reach the objective of our theoretic game approach, we organized the rest of this paper as

follows: in section 2, we present works established concerning the competition over visibility in social networks. In section 3, we propose the study and analysis of the non-cooperative game between selfish and competitive information providers, this analysis consist of employing algorithmic game theory especially the algorithm of Best Response converging to the Nash equilibrium. In section 4, we present obtained results illustrating and assisting our study. Finally, we sum up with a general conclusion opening the window for future works.

2. Related Works

Over the years, social networks are increasingly attracting the attention of academic researchers who tried to present analysis in order to solve specific problematics and model them using various methods and tools.

Concerning the objective of this paper, a previous paper is established by (A. Reiffers-Masson, Y. Hayel, E. Altman, 2014) [13]. This work was basically about presenting an approach to model the competition over visibility in social networks employing the famous game theory.

In order to accomplish this study, authors aimed to model the problem of selecting the content creation's rates as a non cooperative game. They analyzed and reduced it to the well known generalized Kelly mechanism allowing to define and extract the equilibrium between various sources taking into account several groups and the possibility for a message to be relayed from one group to another, but also, authors proved the existence of this equilibrium.

Furthermore, many researches have been conducted in applying the game theory in social networks, these works were basically about studying and modeling the behavior of sources to reach a particular objective [10,11,12].

3. Competition modeling

This section is reserved to model the interaction among information providers (IP), sharing some common destination to which they send contents or post messages, as a non-cooperative game.

In other words, sending new posts makes the older ones desperate from the timeline and pushes sources who are assumed to be rational to resend their posts and share more messages. Further, this behavior is producing a competition concerning the visibility on the walls of social networks.

This problematic leads us to think about the theory of concave games which seems to be the efficient tool for analyzing this kind of competition situations.

In order to attend this objective, we have to specify the number of players, the possible actions for each player, constraints which may be imposed on them and the objective function of each player (source). In fact, each source chooses the appropriate strategy (depending on other sources' strategies) to guarantee its visibility on a feed news. It has also a cost to send messages; this cost depends on its own frequency.

In this context, we consider a system with K information providers. Let λ_j and λ_{-j} be, respectively, the publishing frequencies of a player j and other players. We assume either that γ_j is the cost a player j has for sending posts on a wall of a social network.

Let $G = [K, \{\lambda_j\}, \{U_j(\cdot)\}]$ denotes the non-cooperative frequency game, where K identifies a number of Information Providers, λ_j is the frequency strategy set of IP_j , and $U_j(\cdot)$ is the utility function the IP_j has to maximize by choosing a convenient frequency λ_j .

Formally, the source's j objective function is defined as follows:

$$U_j(\lambda_j, \lambda_{-j}) = K \frac{\lambda_j}{\sum_i \lambda_i} - \lambda_j \gamma_j \quad (1)$$

Where $\lambda_{-j} = (\lambda_1, \dots, \lambda_{j-1}, \lambda_{j+1}, \dots, \lambda_K)$ is the strategy vector of the other IPs.

Assumption 1

To achieve the analysis, we adopt the following assumption:

$$\lambda_j \geq 1, \quad \forall j \in K$$

This hypothesis means that each source publish at least once, which is realistic, because sources that do not publish will not be taken into account.

3.1. The Nash Equilibrium

Considering the competition among selfish IPs, the natural solution of this non-cooperative game will be given by the Nash equilibrium which is considered as a strategy profile such that no provider can unilaterally increase his revenue. Exploring the concave games' theory, we are going to demonstrate the existence and the uniqueness of the Nash equilibrium [9]. We recall that a non-cooperative game G is called concave if all players' utility functions are strictly concave with respect to their corresponding strategies [9].

According to [9], a Nash equilibrium exists in a concave game if the joint strategy space is compact and convex, and the utility function that any given player seeks to maximize is concave in its own strategy and continuous at every point in the product strategy space.

Definition 1

A frequency vector $\lambda^* = (\lambda_1^*, \dots, \lambda_K^*)$ is a Nash equilibrium if for every $j \in \{1, \dots, K\}$,

$$U_j(\lambda_j^*, \lambda_{-j}^*) = \max_{\lambda_j \in [1, \lambda_j]} U_j(\lambda_j, \lambda_{-j}^*)$$

Where $\lambda_{\max}$ is the maximum frequency.

In other way, reaching the equilibrium point, no source could get a profit by changing its strategy in an unilateral manner.

Theorem1

For the game G which is concave the Nash equilibrium exists and it's unique.

Proof 1

To prove the existence of the Nash equilibrium, it is sufficient to show the negativity of the second derivative of U_j ($\forall j$).

$$\frac{\partial U_j(\lambda_j, \lambda_{-j})}{\partial \lambda_j} = K \frac{\sum_i \lambda_i - \lambda_j}{(\sum_i \lambda_i)^2} - \gamma_j$$

This produces the second derivative, given as follows:

$$\frac{\partial^2 U_j(\lambda_j, \lambda_{-j})}{\partial \lambda_j^2} = -2 \frac{K}{(\sum_i \lambda_i)^2} (1 - \frac{\lambda_j}{\sum_i \lambda_i}) \quad (2)$$

The utility function is concave if the result obtained in equation (2) is strictly negative. To ensure this concavity, we have to verify the following condition:

$$1 - \frac{\lambda_j}{\sum_i \lambda_i} > 0$$

According to the assumption 1, $\lambda_j \geq 1$. So the last condition is verified and we obtain the following result:

$$\frac{\partial^2 U_j(\lambda_j, \lambda_{-j})}{\partial \lambda_j^2} < 0 \quad (3)$$

We finally conclude that the game we formulated is concave and this concavity ensures the existence of the Nash Equilibrium.

Now we turn to discuss the uniqueness of the existent Nash Equilibrium, so we must calculate the second derivative of U_j with respect to λ_j and λ_{-j} and verify its test as follows:

$$\frac{\partial^2 U_j(\lambda_j, \lambda_{-j})}{\partial \lambda_j^2} < \sum_{l \neq j} \frac{\partial^2 U_j(\lambda_j, \lambda_{-j})}{\partial \lambda_j \partial \lambda_l} \quad (4)$$

Taking into consideration the U_j defined in equation (1), we compute its first derivative as follows:

$$\frac{\partial U_j(\lambda_j, \lambda_{-j})}{\partial \lambda_j} = -K \frac{\lambda_j}{(\sum_i \lambda_i)^2}$$

So, the second derivative is determined to be:

$$\begin{aligned} \frac{\partial^2 U_j(\lambda_j, \lambda_{-j})}{\partial \lambda_j \partial \lambda_l} &= -\frac{K}{(\sum_i \lambda_i)^2} (1 - 2 \frac{\lambda_j}{\sum_i \lambda_i}) \\ &= -\frac{2K}{(\sum_i \lambda_i)^2} (\frac{1}{2} - \frac{\lambda_j}{\sum_i \lambda_i}) \end{aligned}$$

Or

$$-\frac{2K}{(\sum_i \lambda_i)^2} (\frac{1}{2} - \frac{\lambda_j}{\sum_i \lambda_i}) = -2 \frac{K}{(\sum_i \lambda_i)^2} (1 - \frac{\lambda_j}{\sum_i \lambda_i}) + \frac{K}{(\sum_i \lambda_i)^2}$$

That produces

$$\frac{\partial^2 U_j(\lambda_j, \lambda_{-j})}{\partial \lambda_j \partial \lambda_l} = \frac{\partial^2 U_j(\lambda_j, \lambda_{-j})}{\partial \lambda_j^2} + \frac{K}{(\sum_i \lambda_i)^2}$$

Therefore, the sum of these derivatives is obtained as:

$$\sum_{l \neq j} \frac{\partial^2 U_j(\lambda_j, \lambda_{-j})}{\partial \lambda_j \partial \lambda_l} = \sum_{l \neq j} (\frac{\partial^2 U_j(\lambda_j, \lambda_{-j})}{\partial \lambda_j^2} + \frac{K}{(\sum_i \lambda_i)^2}) \quad (5)$$

Or the equation (3) shows that the second derivative of the utility function is strictly negative, $\forall \lambda_j \geq 1$.

And

$$\frac{K}{(\sum_i \lambda_i)^2} > 0$$

Using the result reached in equation (5), we prove the test presented in equation (4). Hence, the uniqueness of the Nash equilibrium is guaranteed.

3.2. Convergence to the Equilibrium: Algorithm of the Best Response

The demonstration of the existence of a unique Nash equilibrium leads to the convergence study using the algorithmic game theory, a fascinating fusion of ideas of both fields -game theory and algorithms-, [2], as algorithms became the natural environment and default platform of strategic decision making. The particular task of this work is to employ it in order to learn the equilibrium's parameter. Algorithm defines a best response sequence as a couple $(t(n), j(n))$ where $t(n)$ is a strictly increasing sequence of times such that at time $t(n)$ player $j(n)$ updates its action using a best response to the current actions of the other players. Therefore, there is a unique equilibrium and any sequence of best responses of the players converges to it in a finite time.

As shown in equation (3), the utility functions $U_j(\lambda_j, \lambda_{-j})$ are concave. So, for each user j , the frequency maximizing its utility is the solution of the following equation:

$$\frac{\partial U_j(\lambda_j, \lambda_{-j})}{\partial \lambda_j} = 0$$

The resolution of the last equation introduced the best response in term of frequency $BR_{\lambda_{-j}}^j$ of each source j as a function of its opponents' strategies. We define this notion as following:

$$BR = \operatorname{argmax}_{\lambda_j} U_j(\lambda_j, \lambda_{-j})$$

As already seen, a sequence of best responses converges to the Nash Equilibrium since this one is unique. Algorithm 1 details the work done using the BR [8].

Algorithm 1: Best Response

Initialization of the frequency vector λ ;

For each source $j \in K$ at iteration t :

$$\lambda_j^{t+1} = BR_{\lambda}^j(\lambda^t)$$

This algorithm aims to compute the best response that maximize the utility function. When the $BR_{\lambda}^j(\lambda^{t-1}) = BR_{\lambda}^j(\lambda^t) = BR_{\lambda}^j(\lambda^{t+1})$, we conclude that the algorithm has converge to the equilibrium point.

4. Results and discussions

In this paper, we report on what we consider a noticeable application of game theory to model competition over visibility in social networks. To clarify our analysis, we suggest to study numerically the equilibrium frequency λ^* by considering the utility functions of sources as well as the best response of each one. Hence, we consider a social network with two sources $K=2$ trying to maximize their respective revenues in term of visibility in this system. The frequency strategies' set is assumed to be $\lambda_j = [1, 80]$ and the parameter γ_j , which is the indicative of the cost paid by the competitors to operate at higher frequencies, is the same for both sources.

The specification of the cost's value must take into consideration that the utility function has to be positive for each set of frequencies.

$$U_j(\lambda_j, \lambda_{-j}) > 0, \forall \lambda_j \in [1, 80]$$

Assuming that $\lambda = (\lambda_j, \lambda_{-j}) = (15, 1)$ maximizes the U_j , the cost we're searching for is the solution of the following equation:

$$\frac{\partial U_j(\lambda_j, \lambda_{-j})}{\partial \lambda_j} = 0$$

It gives that:

$$K \frac{\sum_i \lambda_i - \lambda_j}{(\sum_i \lambda_i)^2} - \gamma_j = 0$$

Replacing the frequencies with their values, we obtain the cost's value which is:

$$\gamma_j = 2 * \left(\frac{1}{(16)^2} \right) = \frac{2}{256}$$

Taking into account that the named players have a symmetric behavior, in the rest of this section, we will study their visibility with respect to the different parameters of the considered system and present the obtained results only for one source hence it's the same for the other player.

*** Fixed cost $\gamma_j = 2/256$**

Assuming that sending messages on a wall comes with a fixed cost, we firstly plot, in Figure 1, the utility function of the player j considering that λ_i ; the other player's frequency; is equal to 1 ($\lambda_i = 1$).

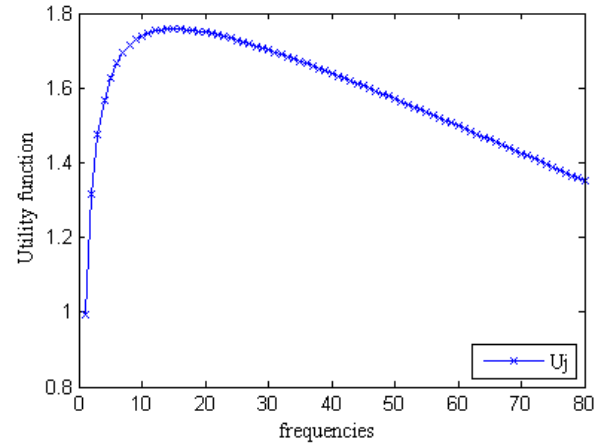


Figure 1: Function utility of player j when $\lambda_i = 1$

According to the graph, we remark that U_j is a concave function. Henceforth, the existence and uniqueness of the equilibrium situation are guaranteed. We will thus present the best responses for both sources in Figure 2

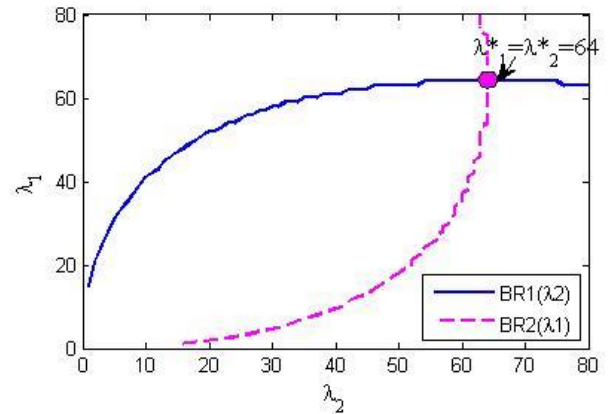


Figure 2: Best responses for both sources

The presentation of the BRs reveals a point of intersection; this point constitutes a point of equilibrium among the considered sources. It is numerically showed to be $\lambda^*_1 = \lambda^*_2 = 64$.

*** Cost $\gamma_j = [2/256, 50/256]$**

In this part, we'll compute the frequency's sets at the equilibrium and present the utility functions of each source j with respect to several values of posting messages' cost.

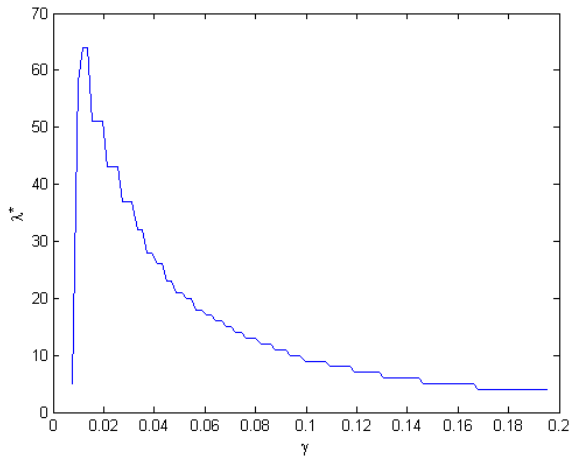


Figure 3: Evolution of λ at the equilibrium WR to γ_j

According to the graph presented in Figure 3, we conclude that the algorithm employed to accomplish our work has converged to the equilibrium for each value of the cost γ_j . We remark also that when the cost increases the set of frequencies assuring the equilibria knows a decreasing evolution.

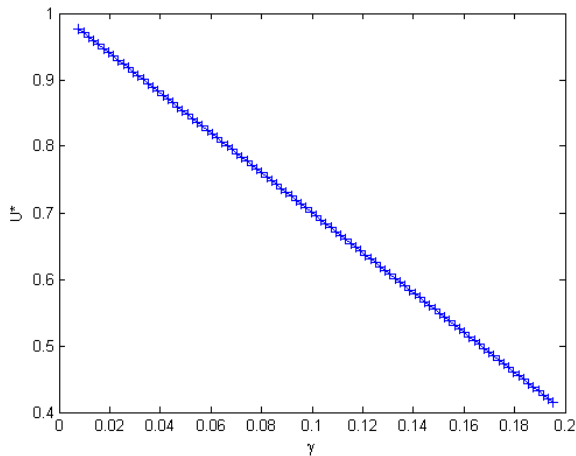


Figure 4: Utility functions at the equilibrium WR to γ

The figure 4 reveals the values of utility functions for each source, these values has an evolution in a decreasing manner with respect to the cost. The evolution of U_j follows the decreasing values of the frequencies' set obtained at the equilibrium.

To generalize the reached results of this work, we take into consideration that inside a system constituted by K sources, the interaction is the same between each couple of sources. Henceforth, the equilibrium exists and is unique in a game with K sources and the obtained results assure the aim of our theoretical approach.

5. Conclusions

In this paper, we were concerned with a situation where many participants interact; a system where sources try their best and compete to keep their visibility on timelines. Giving that such a situation is naturally modeled by games that involve many players, we aimed to analyze this sort of competitions as a non-cooperative game. We took into consideration that the utility function of a user depends on his frequency of sending posts on the wall of a social network but also on other users' frequencies, and posting messages comes with a cost which is proportional for its frequency. We formulated the named game using the theory of concave games to which the Nash equilibrium seemed to be the efficient solution. Concerning the numerical investigations, we considered a social network containing 2 sources competing over visibility, we explored the best response algorithm to compute the equilibrium which is found in a finite number of iterations and present the results obtained concerning the frequencies and the utility functions at the equilibrium for each source of information.

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