International Arab Conference on Information Technology (ACIT'2016)

On Spatially Coherent Noise Field in Narrowband Direction Finding

Youssef Khmou, Said Safi
Department of Mathematics and informatics, Sultan Moulay Slimane University, Morocco

Abstract: *When studying the radiation coming from far field sources using an array of sensors, besides the internal thermal noise, the received wave field is always perturbed by an external noise field, which can be temporally and spatially coherent to some degree, temporally incoherent and spatially coherence, spatially incoherent and temporally correlated or finally the incoherence in both domains. Thus treating the received data needs to consider the nature of perturbing field in order to make accurate measurements such as powers of punctual sources, theirs locations and the types of waveforms which can be deterministic or random. In this paper, we study the type of temporally white and spatially coherent noise field; we propose a new spatial coherence function using Lorentz function. After briefly describing some existing models, we numerically study the effect of spatial coherence length on resolving the angular locations of closely radiating sources using spectral techniques which are divided into beam forming and subspace based methods, this study is made comparatively to temporally and spatially white noise with the same power as the proposed one to make a precise conclusions. Finally we discuss the possibility of extending the spatially coherent noise field into two dimensional geometries such as circular array*.

Keywords: *Spatial coherence function, narrowband, direction of arrival, Lorentz function, coherence length, white noise field*.

1. Introduction

Among the applications of narrowband radio communications [1]-[2]-[3], is the treatment of interactions between waves which results in spatial interferences that are related to the notion of coherence [1]-[4]-[5]. This last can be spatial, temporal or both. In fact, given a wave field $E(r,t)$ at instant t and position r, the coherence gives the period and the range where we can predict the characteristics of the field in different spatial and temporal locations $E(r',t')$.

In the majority of applications such as radiolocation and underwater acoustics [6], the coherence of the total electric field and that of perturbing field becomes a problem, because for these applications, the goal is to characterize a wave field by separating the components of each elementary source such as the spatial position (r,θ,φ) and power σ^2. If the total electric field is totally or partially coherent, then some pre-processing treatments are mandatory to decorrelate the sources [7]. In order to explain this process of decorrelation, let us take an example of simple configuration consisting of an array of omnidirectional receiving sensors and two transmitting sources which are closely spaced and temporally coherent, a simple processing of the received data by the array indicates that the bearing is originated from one source, therefore, to make a correct analysis, the decorrelation is necessary which is based on some statistical computing. In other cases, the emitting sources can be temporally uncorrelated, but the perturbing field intercepted by the array can be spatially coherent, the noise field can be localized in the vicinity of the array or made in the far field [8]. The power and the spatial coherent length of this interfering field have an impact on result analysis; they can degrade the resolution ability and the interpretation of the obtained results.

As the coherence is characterized by temporal and spatial parameters, the four possible cases are: spatial and temporal coherence which is the case for an almost ideal monochromatic wave, spatial incoherence and temporal coherence, spatial and temporal incoherence, finally the case of temporal incoherence and spatial coherence, in this study we are focused on the last possibility. In terms of statistical description, the third case is said to be temporally and spatially ergodic process in wide sense, it is a realization of white noise [9] described by Gaussian probability distribution. For the fourth possibility, if we make measurements in different positions, we find a pattern of spatial coherence which can be total or partial (degree of correlation). In the literature, the spatial coherence functions have, in general, the same property, indeed, if we measure the powers between several points r_i, we find that the pattern decreases [10]-[11]-[12], and may become zero for large distance $r_i - r_j$.

In the other cases, given that the sensors are designed to function with defined frequency ranges, then the distance between elementary sensors have an impact of the noise field, in fact, the coherence can be stronger if the distance between consecutive sensors is strictly less than half of the central wavelength λ [12] that corresponds to the operational frequency of the array. Based on these observations, many modeling functions have been proposed [13]-[14] to study the effect of coherence on spatial-spectral analysis of the radiating sources.

Our objective in this study is to propose a new spatial coherence function for an array of identical and omnidirectional sensors operating with the same carrier frequency as that of punctual and far field sources, we model the spatial pattern of coherence by Lorentz function [15] and we discuss some simulation results of this model in terms of spatial accuracy of some high resolution angle of arrival techniques.

This paper is organized as the following, in literature review section, we describe the data model according to uniform linear array in the context of far field radiation and we present a brief description of some models of spatial coherence, in the third section, we propose a new spatial coherence function using Lorentz function.

Next, we discuss in the fourth section the results of Monte Carlo simulation for closely sources. Finally we extend the proposed function into two dimensional geometry of sensors.

2. Literature review

We consider a uniform linear array of N omnidirectional sensors separated by distance d and described by position vector $r = [r_1 = 0, r_2 = d, \dots, r_{N-1} = (N-1)d]$, the far field contributions of P radiating sources create a wavefield that we suppose is linearly polarized $E = [0,0,E_z]$, the expression of the total electric field at r_j measuring point is given by

$$E_z(r_j,t) = \sum_{i=1}^{P} s_i(t) e^{i(\omega t - \vec{k}_i . \vec{r}_j)} \qquad (1)$$

Where $\omega = 2\pi f$ is the angular frequency, $\vec{r}_j = (j-1)d$ and $s_i(t)$ is the i^{th} slowly varying envelope [2], $\vec{k}_i$ is the i^{th} wave vector [2] defined in spherical coordinates by:

$$\vec{k}_i = \frac{-2\pi}{\lambda} \begin{pmatrix} \cos\theta_i \sin\varphi_i \\ \sin\theta_i \sin\varphi_i \\ \cos\varphi_i \end{pmatrix} \qquad (2)$$

λ is the wavelength which is approximately the same for all P sources and (θ_i, φ_i) are the i^{th} angle of arrival (azimuth) and elevation respectively. For simplicity, we consider only two dimensional description by fixing $\varphi_i = \frac{\pi}{2}$. If we suppose that the envelopes of sources are uncorrelated, then the temporal coherence at position r_i of the wavefield is given by :

$$< E_z(r_j,t) E_z^*(r_j,t+\tau) >= \sum_{i=1}^{P} \sigma_i^2 \delta(\tau) \qquad (3)$$

Where $<.,.>$ is the time average operator, σ_i^2 is the power of the i^{th} waveform $s_i(t)$ and $(.)^*$ is the conjugate operation. In the presence of interfering noise $E_n(r,t)$, then the total intercepted wavefield by the array can written as :

$$E(r,t) = E_z(r,t) + E_n(r,t) \qquad (4)$$

At the reception point r_j, the induced voltage by $E(r_j,t)$ can be written, after down conversion and sampling, as the following :

$$x_j(t) = \sum_{i=1}^{P} s_i(t)\, e^{-i2\pi\lambda^{-1}\sin(\theta_i) r_j} + n_j(t) \qquad (5)$$

Where we have K samples $t = 1, \dots, K$,given that N channels are available, the data is written in vector form as :

$$x(t) = a(\theta)s(t) + n(t) \quad (6)$$

$x(t) \in \mathbb{C}^{N\times 1}$ and $a(\theta) \in \mathbb{C}^{N\times P}$ is the steering matrix [2] that depends on the geometry of the array $a(\theta) = [a_1(\theta), \ldots, a_P(\theta)]$, the i^{th} steering vector is given as :

$$a_i = \left[1, e^{-i2\pi\lambda^{-1}\sin(\theta_i)d}, \ldots, e^{-i2\pi\lambda^{-1}\sin(\theta_i)(N-1)d}\right]^T \quad (7)$$

$s(t) \in \mathbb{C}^{P\times 1}$ is the envelope vector at instant t and $n(t) \in \mathbb{C}^{N\times 1}$ is noise vector which is generally described as white noise, zero mean ergodic process [9] that verifies the following relations :

$$< n_i(t)n_i^*(t+\tau) >= \sigma^2\delta(\tau)$$

$$< n_i(t)n_j^*(t) >= \sigma^2\delta_{ij}$$

$$< s_i(t)n_j^*(t) >= 0 \quad (8)$$

$$< n(t)n^+(t) >= \sigma^2 I_N$$

σ^2 is the uniform power for N channels, $(.)^+$ is the conjugate transpose operator and I_N is the identity matrix. Based on relations in equation (8), the spatial correlation matrix is defined by :

$$\Gamma =< x(t)x^+(t) >= a < s(t)s^+(t) > a^+ + \sigma^2 I_N$$

$$= \Gamma_s + \Gamma_n \quad (9)$$

The majority of arrival angle estimation spectral techniques are based on Γ to locate the directions of propagation [2]-[9], theirs spatial resolutions depend on many factor such as the statistical correlation of noise field $n(t)$. As we mentioned earlier, we are focused on spatial coherence, if $n(t)$ has correlation pattern that depends on array geometry, some angle of arrival estimation techniques may be degraded in their resolution power [16]. In the case of spatial coherence, the noise field operator Γ_n is not diagonal but banded matrix, for example in [11], the spatial correlation is described by the operator:

$$\Gamma_n(u,v) = \sigma^2 \rho^{|u-v|} \quad (10)$$

for parameter $0 \le \rho \le 1$, we can remark that in case $\rho = 0$, Γ_n corresponds to white noise.

Another model of spatially correlated noise is given by the operator [17]:

$$\Gamma_n(u,v) = \begin{cases} \sigma^2 \rho^{|u-v|} e^{i\pi(u-v)/2} \\ 0 \; if \; |u-v| > l \end{cases} \quad (11)$$

where l is the spatial correlation length such as $l < N$.in the second example we can also remark that if the parameter ρ tends to zero then Γ_n described the correlation of spatially ergodic noise.

Another well known model for spatially correlated noise field is the spherically isotropic model [13]-[12], where the field received by the array is coming from sources uniformly distributed on sphere surrounding the array where the radius is much larger than the array length $D = (N-1)d$. The noise operator is given by the following expression:

$$\Gamma_n(u,v) = \sigma^2 \frac{\sin(k|u-v|)}{k|u-v|} \quad (12)$$

Where k is the wavenumber $k = 2\pi/\lambda$. this model is used for distances less than half of the wavelength, because we can remark that for an N uniform linear of $d = \lambda/2$, the operator describes the case of spatially white noise $\frac{\sin(knd)}{(knd)} = 0$for $n = 0, \ldots, N-1$. Based on these models, we present in the next section a new spatial correlation pattern using Lorentz function.

3. Proposed spatial coherence function

In this section, we present our model to describe the spatial coherence of noise field captured by the array which is a function of noise power and spatial coherence length l, the model is based on the following assumptions.

- The field is characterized by uniform power on all measurement points σ^2.
- The field is temporally ergodic, thus we have $< n_i(t)n_i^*(t+\tau) >= \sigma^2\delta(\tau)$.
- Given σ^2 on i^{th} sensor, the field is said to be uncorrelated at j^{th} sensor if the interacting power does not exceed 10% of σ^2, therefore the spatial coherence length is defined as $l = |r_j - r_i|$.

- Given the noise field $n(t) \in \mathbb{C}^{N \times K}$, the real and imaginary parts follow the same model.

The spatial coherence function is version of Lorentz function [15], for distance $|r_j - r_i|$, we have :

$$f(r_i, r_j, \sigma^2, \beta) = \frac{\sigma^2}{\beta(r_j - r_i)^2 + 1} \qquad (13)$$

Where β is a parameter that controls the width which must be a function of l. If we measure the power, for linear array, at r_i where $f(r_i, r_i, \sigma^2, \beta) = \sigma^2$, the correlation at position r_j is negligible if $f < 0.1\sigma^2$ such as the spatial correlation length is $l = |r_j - r_i|$, from this assumption, the width parameter is given by the following criterion :

$$\beta = \frac{9}{l^2} \qquad (14)$$

In this case, the full width at half maximum is $FWHM = 2/\sqrt{\beta}$. To study the coherence as function of distance, we compare the proposed function with spherically isotropic noise and exponential models as represented in Figure 1.

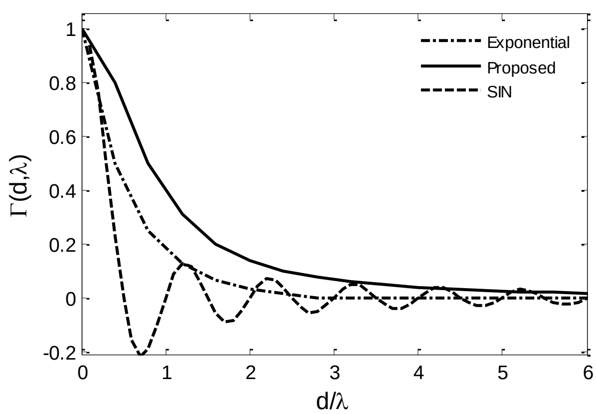

Figure 1. Comparing three different models of spatially coherent noise field.

The parameters of the proposed model are $d = 0.4\lambda$, $l = 6d$ and $\sigma^2 = 1$ W, for the exponential model, the parameters are $\rho = 0.5$ and $l = 6d$. We can remark that the Lorentz model is an envelope approximation of the spherical model.

After characterizing the function, we present the computational method to generate the complex data $n'(t)$, received by the array, which is temporally white and follow the spatial Lorentz function. The inputs are the number of sensors N, the distance as function of wavelength $d(\lambda)$, the spatial coherence length l, the uniform power σ^2 and the number of digital samples K. The outputs are the realization of the noise field matrix $n'(t)$, the theoretical and estimated correlation matrices Γ_{th} and Γ_n.

- Inputs: N, l, $d = f(\lambda)$, σ^2, K and $\beta = 9/l^2$.
- For $u = 1{:}N$ and $v = 1{:}N$, compute :
$$\Gamma_{th}(u, v) = \frac{\sigma^2}{\beta(d(u - v))^2 + 1}$$
- Generate $n_0(t) = \frac{1}{\sqrt{2}}(a(t) + jb(t))$, where $a(t), b(t) \sim \mathcal{N}(0_{N\times 1}, I_N)$ and $t = 1, \dots, K$.
- Generate $n'(t) = \Gamma_{th}^{1/2} n_0(t)$.
- Estimate the correlation matrix by the relation $\Gamma_n = n'(t)n'^{+}(t)/K$.

We remark that the theoretical correlation matrix is real $\Gamma_{th} \in \mathbb{R}^{N \times N}$ while the estimated is complex valued $\Gamma_n \in \mathbb{C}^{N \times N}$, and both are hermitian $\Gamma_{th}^{+} = \Gamma_{th}$, $\Gamma_n^{+} = \Gamma_n$. The matrix Γ_n is accurately estimated if the number of samples K is sufficiently large, the quality of estimation can be verified by computing the Root Mean Square Error of eigenvalues as:

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(\lambda_{th,i} - \lambda_i)^2} \qquad (15)$$

Where $\lambda_{th,i}$ are the eigenvalues of Γ_{th} and λ_i are those of Γ_n. As previously discussed for other models of spatially coherence noise, this model also converges to spatially ergodic noise if the spatial coherence length l tends to zero :

$$\lim_{l \to 0} \Gamma_n = \sigma^2 I_N \qquad (16)$$

After generating the new matrix $n(t)$, the signal model of the array becomes $x(t) = a(\theta)s(t) + n'(t)$ where the parameter l has in impact on the accuracy of estimating the directions of the P propagating waves using spectral techniques [16], this relationship is discussed in the next section.

4. Results and discussion

Given the Lorentz model of spatially coherence noise field, we run in this part, some computer simulation to verify the proposed formalism, and test the resolution power of direction of arrival techniques [2]-[9] for specific value of spatial correlation length l.

In the first part, we simulate an array consisting of $N = 18$ omnidirectional sensors where the gain of each sensor is $g_i(\theta) = 1$. The array is operating with one carrier frequency $f = c/\lambda$ and the distance between sensors is set to $d = 0.4\,\lambda$. We simulate the spatially coherent noise field $n'(t)$ using the steps described in the previous section with parameters $\sigma^2 = 0.1$ W, $l = 12d$ and $K = 300$ samples, the theoretical spatial coherence function for the $nine^{th}$ sensor comparatively to the estimated one are represented in Figure 2.

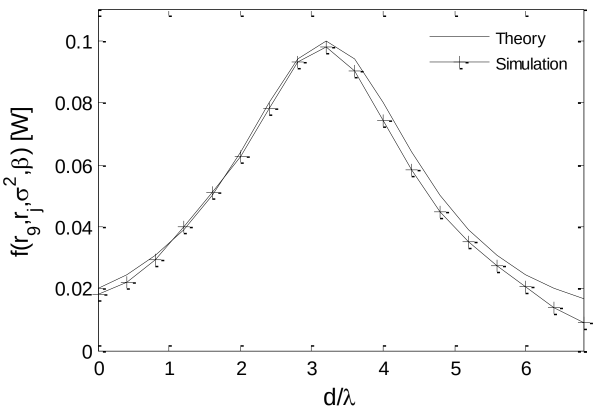

Figure 2. spatial coherence function of the central element in the array with $N = 18$, $l = 12d$ and $d = 0.4\,\lambda$, theory and simulation.

In this trial, the Root Mean Square Error of eigenvalues is $RMSE = 0.0154$ W. In the second part, we begin the analysis of radiating sources where we consider the position of the array as the origin of the referential. We assume the presence of $P = 3$ far field and punctual sources that are producing radiations with approximately the same frequency of the array f.

These sources are characterized by linear polarization $E_x = E_y = 0$ and are located in the same horizontal plan with (x, y) coordinates given by $(100\,\lambda, 38.39\,\lambda)$, $(80\,\lambda, 35.61\,\lambda)$ and $(70\,\lambda, 38.80\,\lambda)$ as depicted in Figure 3.

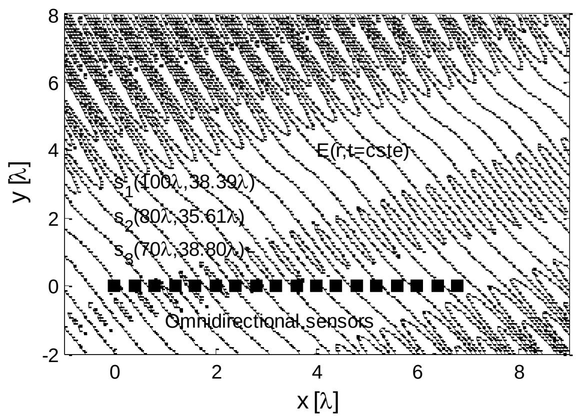

Figure 3. Simulation of noise-free wave field consisting of three sources in the far field region.

The locations of the sources correspond to angles of arrival $\theta = [21°, 24°, 29°]$ which we use to generate the steering matrix $a(\theta)$ and the signal model $x(t)$ in the presence of spatially coherent noise field.

Given the array length $D = (N-1)d = 6.8\,\lambda$, Rayleigh limit of angular resolution of this array is approximately equal to $\theta_{HPBW} \cong \lambda/dN \cong 8^\circ$, therefore estimating the angles of arrival requires high resolution methods, before discussing these spectral techniques, we study in the third part, the estimation of Γ_n of spatially correlated noise from $x(t)$ where the type of waveforms is $s(t) \sim \mathcal{CN}(0_{P\times1}, I_P)$.

The spatial correlation matrix $\Gamma = x(t)x^+(t)/K$ can be decomposed into $\Gamma = U\Lambda U^+$ where $U \in \mathbb{C}^{N\times N}$ is unitary matrix and $\Lambda \in \mathbb{R}^{N\times N}$ is diagonal matrix where the diagonal elements are the eigenvalues given by $\{\lambda_1, \ldots, \lambda_P, \lambda_{P+1}, \ldots, \lambda_N\}$[3].

The first P eigenvalues correspond to the signal subspace $U_s \in \mathbb{C}^{N\times P}$ and the remaining eigenvalues belong to the noise subspace $U_n \in \mathbb{C}^{N\times N-P}$ [3] such as $U = [U_s, U_n]$.

In the case of temporally and spatially ergodic noise, the projector into the noise subspace $P_n = U_n U_n^+$ is orthogonal to the steering matrix $a(\theta)$:

$$P_n a(\theta) = 0_{N\times P} \tag{17}$$

From this property, it is possible to estimate the operator Γ_n using the following equations:

$$F = P_n x(t) = P_n a(\theta)s(t) + P_n n(t) \cong P_n n(t)$$

$$\hat{n}(t) = P_n^{\dagger} F \tag{18}$$

$$\hat{\Gamma}_n = \hat{n}(t)\hat{n}^{+}(t)/K$$

Where $(.)^{\dagger}$ is the generalized inverse operator, using these steps, we compare the estimation of ergodic noise $n(t)$ and coherence model $n'(t)$ with respect to power while the powers of the source are kept at $\sigma_i^2 \cong 1$ W, the Signal to Noise Ratio is defined by the relation $SNR = 10log_{10}(1/\sigma^2)$ which varies from -10 dB to 20 dB where, for $L = 20$ trials of each value of SNR, we compute the estimate $\hat{\Gamma}_n$ for both noise models using equations (18), we measure the efficiency of estimation by average power of the error $|\hat{\Gamma}_n - \Gamma_n|$ defined by the relation :

$$< e > = \frac{Tr(|\hat{\Gamma}_n - \Gamma_n|)}{N} \tag{19}$$

where $Tr(.)$ is the trace operator, the results of the estimation are given in Figure 4.

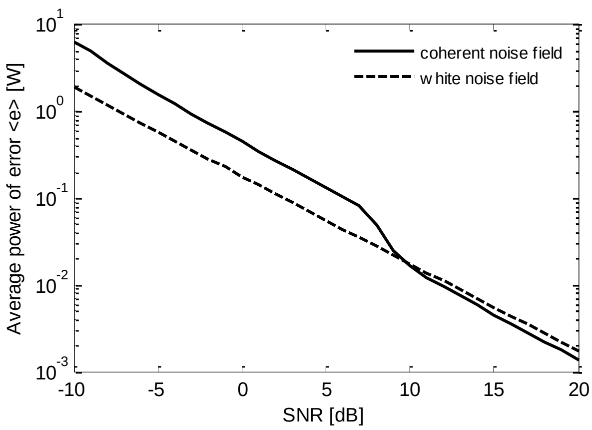

Figure 4. Average power of error $< e >$ between original and estimated noise field matrices, for different values of SNR.

For level $SNR \leq 8$ dB, the average error of spatially coherence noise field with $l = 12d$ is higher than that of white noise, staring from 8 dB, estimating the noise coherence matrix Γ_n is more accurate.

From this result we fix the noise level at $SNR = 10$ dB for the fourth part of the simulation, the number of Monte Carlo trials is $L = 200$ and the angular region of searching is set to $\Omega = [15°, 40°]$ with angular step of $d\theta = 0.1°$. for each value of θ we generate the steering vector a $a \in \mathbb{C}^{N\times 1}$ by which the localization function [9] is given by :

$$f(\theta \in \Omega) = \frac{1}{a^{+}P_n a} \tag{20}$$

$P_n \in \mathbb{C}^{N\times N}$ is chosen spectral operator. We compare the resolution power of average localization function, over L trials, for Minimal Variance Distortionless Response operator (MVDR) [18] and Maximum Entropy Method (MEM) [9] as shown in Figure 5 and Figure 6.

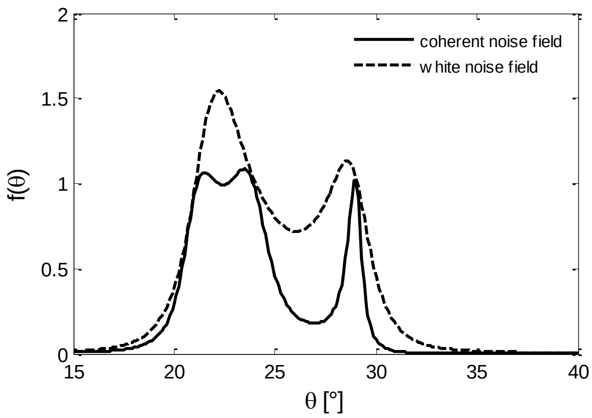

Figure 5. MVDR localization function, $SNR = 10$ dB, $l = 12d$.

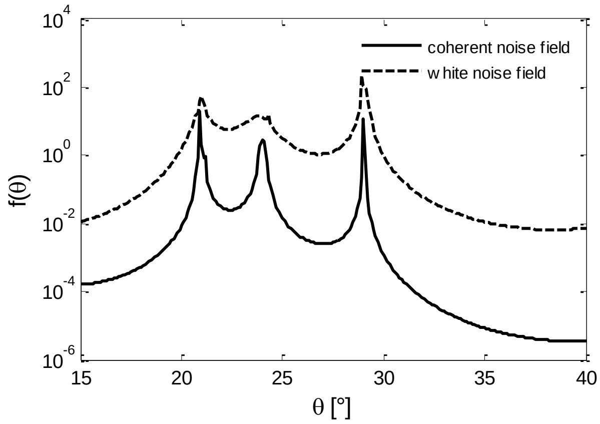

Figure 6. MEM localization function, $SNR = 10$ dB, $l = 12d$.

We can remark that the MVDR technique in the case of spatially coherent noise field has better resolution ability, which is also the case for MEM operator that is computed using the first column of inverse correlation matrix Γ^{-1}. Next, we implement the Orthonormal Propagator Method (OPM) [19] and Minimum Norm Method (MIN) [20], as shown in Figure 7 and Figure 8.

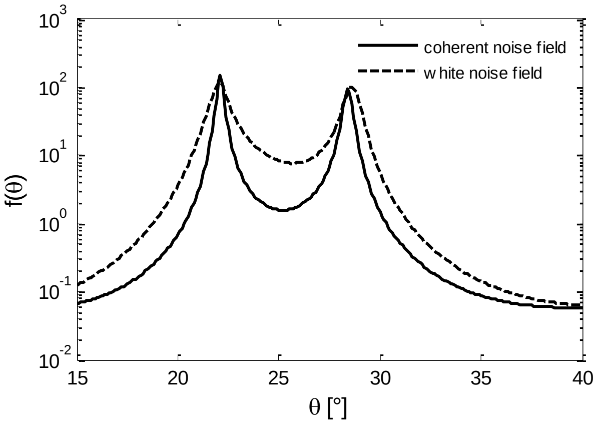

Figure 7. OPM localization function, $SNR = 10$ dB, $l = 12d$.

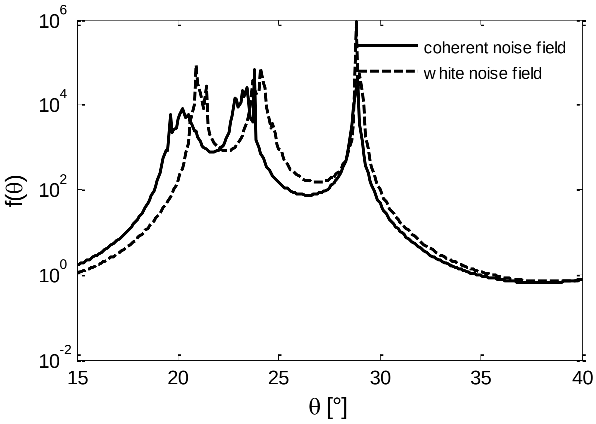

Figure.8. Min Norm localization function, $SNR = 10$ dB, $l = 12d$.

The OPM localization function cannot separate the sources at $\theta_1 = 21°$ and $\theta_2 = 24°$ for both noise fields, the MIN Norm function has enhanced resolution power however the fluctuations of peaks in $f(\theta)$ make the interpretation difficult to make.

In Figure 9, we present of the spectrum of Generalized Maximum Entropy Method G-MEM, which is defined by the operator:

$$P_n = \sum_{i=1}^{N} \frac{C_i C_i^+}{Tr(C_i C_i^+)} \tag{21}$$

Where $C_i \in \mathbb{C}^{N\times 1}$ is the i^{th} column of inverse correlation matrix Γ^{-1}. G-MEM operator has a better resolution ability in this case of Lorentz noise field. Next, we compare the Multiple Signal Classification technique (MUSIC) [3] in Figure 10 where the localization function is not successful in the case coherent noise field because the closely sources are not separated.

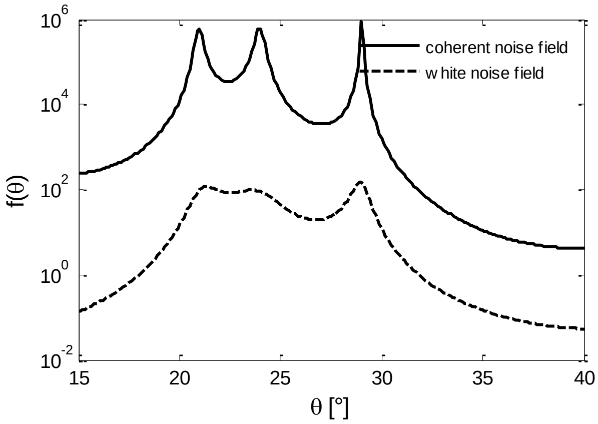

Figure 9. G-MEM localization function, $SNR = 10$ dB, $l = 12d$.

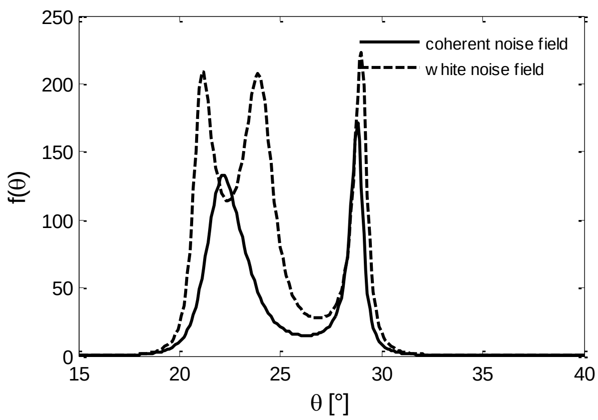

Figure 10. MUSIC localization function, $SNR = 10$ dB, $l = 12d$.

Finally we compare Pisarenko Harmonic Decomposition method (PHD) [9] and Lorentzian operator, a recently proposed technique [21], given in Figure 11 and Figure 12.

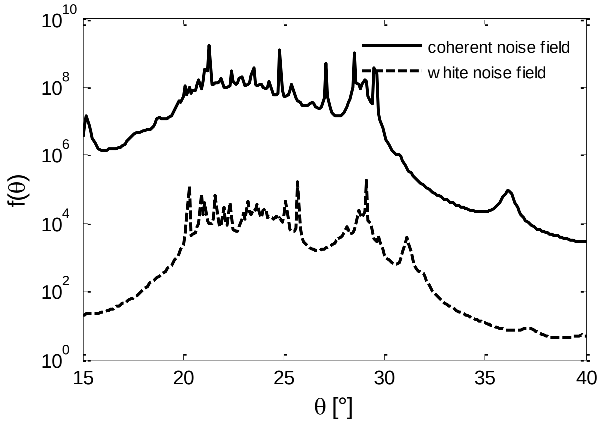

Figure 11. PHD localization function, $SNR = 10$ dB, $l = 12d$.

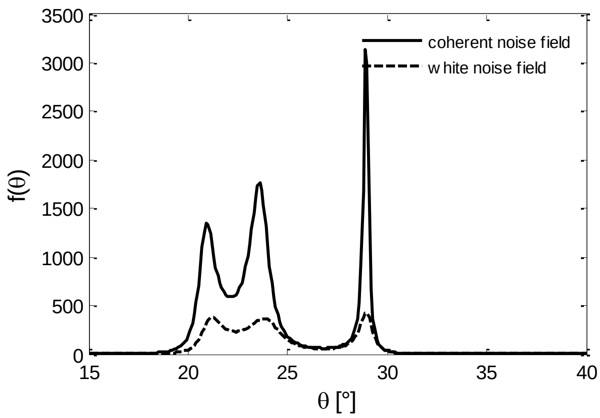

Figure 12. Lorentz localization function, $SNR = 10$ dB, $l = 12d$.

The PHD spectrum is computed using the eigenvector corresponding to the smallest eigenvalue λ_N. In this case, the localization function contains noisy peaks for both noise fields. The Lorentzian function has an increased resolution power for spatially coherent noise field.

We conclude from these simulation results that in moderate SNR conditions (10 dB) and for large number of sensors $N = 18$, the beamforming based techniques have a better resolution power in the case of spatially coherence noise field, comparatively to the subspace based techniques when the spatial coherence length l is almost 70 % of array's length D.

As perspective of this study, this proposed model of noise field can be applied to closely sources in the case of near field localization problem, the correlation between the waveforms of sources $< s_i(t)s_j^+(t) >$ becomes a function of distance between i^{th} and j^{th} sources where the spatial parameter l can be considered a function of wavelength λ.

The existing spatially coherent noise field models are generally studied and simulated using one dimensional geometries. If we take the proposed model $f(r_i, r_j, \sigma^2, \beta)$, the correlation operator Γ_n must not have the same structure if we consider the circular geometry of sensors.

Before developing the spatial coherence for two dimensional geometry, we have to make an assumption that the spatially coherent noise field is coming from punctual sources uniformly placed around the array, taking one sensor, the noise field's spatial correlation l must be the same if we take any direction from that sensor, the field is uniform in region $\Omega = 2\pi$ such as no particular direction θ is privileged.

Let us consider a circular array of N sensors and radius r, the distance between i^{th} and j^{th} sensors, denoted by $d_{ij} = (r_j - r_i)$, can be written in polar coordinates by the following relation :

$$d_{ij} = \sqrt{2r^2\left(1 - \cos\left(\frac{2\pi(j-i)}{N}\right)\right)} \qquad (22)$$

The radius is function of wavelength and distance between sensors d, given $d = \mu\lambda$ where in generally we have $\mu \leq 0.5$ ($\mu = 0.4$ in presented simulations), the radius is given by :

$$r = \sqrt{\frac{\mu^2\lambda^2}{2\left(1-\cos\left(\frac{2\pi}{N}\right)\right)}} \qquad (23)$$

Next, the spatial coherence function for circular array is written with new expression of distance d_{ij}, in equation (22), as the following :

$$f(d_{ij}, \sigma^2, \beta) = \frac{\sigma^2}{\beta d_{ij}^2 + 1} \qquad (24)$$

Another perspective consists of testing the performance of direction finding spectral techniques for circular array with different values of radius r in the presence of spatially coherence noise field where the spatial correlation length does not exceed the maximum distance of the array which is $2r$ and make conclusions in comparison with other coherent models.

5. Conclusion

In this paper, we have proposed a new spatial coherence function of noise field received by an array of sensors in the context of narrowband and far field source localization problem where the radiating sources are stationary over the period of observation. In the first part, we have briefly described some existing models of spatially correlated noise, next we have proposed a new model based on Lorentz decreasing pattern. In the second part, we conducted some computer simulations to test the robustness of angle of arrival techniques in the presence of such field, the results proved that for large array, closely sources, moderate perturbation level and 70 % of spatial correlation length with respect to array length, the beam forming techniques have a better resolution power compared to subspace based techniques. In the last part, we have extended the proposed model into circular geometry of sensors.

References

[1] A. B. Gershman, C. F. Mecklenbrauker, and J. F. Bohme. Direction finding with imperfect wavefront coherence: a matrix fitting approach using genetic algorithm. In *Acoustics, Speech, and Signal Processing, 1997. ICASSP-97., 1997 IEEE International Conference on*, volume 1, pages 519–522 vol.1, Apr 1997.

[2] H. Krim and M. Viberg. Two decades of array signal processing research: the parametric approach. *IEEE Signal Processing Magazine*, 13(4):67–94,Jul 1996.

[3] R. Schmidt. Multiple emitter location and signal parameter estimation. *IEEE Transactions on Antennas and Propagation*, 34(3):276–280, Mar 1986.

[4] Tie-Jun Shan, M. Wax, and T. Kailath. On spatial smoothing for direction-of-arrival estimation of coherent signals. *IEEE Transactions on Acoustics,Speech, and Signal Processing*, 33(4):806–811, Aug 1985.

[5] S. Su, Z. Wang, and J. Yan. New method of doa estimation for coherent sources under unknown noise field. In *Electronic Measurement Instruments,2009. ICEMI '09. 9th International Conference on*, pages 4–569–4–572, Aug 2009.

[6] J. Tao, W. Chang, and W. Cui. Vector field smoothing for doa estimation of coherent underwater acoustic signals in presence of a reflecting boundary. *IEEE Sensors Journal*, 7(8):1152–1158,Aug 2007.

[7] X. q. Hu, J. w. Chen, H. Chen, and Y. l. Wang. Estimation doas of the coherent sources based on toeplitz decorrelation. In *Image and Signal Processing,2008. CISP '08. Congress on*, volume 5, pages 54–58, May 2008.

[8] A. Kisliansky, R. Shavit, and J. Tabrikian. Direction of arrival estimation in the presence of noise coupling in antenna arrays. *IEEE Transactions on Antennas and Propagation*, 55(7):1940–1947, July 2007.

[9] Youssef Khmou, Said Safi, and Miloud Frikel. Comparative study between several direction of Arrival estimation methods. *Journal of Telecommunications and Information Technology*, 1:41–48, 2014.

[10] Y. Khmou and S. Safi. Doa estimation with fourth order propagator. In *Multimedia Computing and Systems (ICMCS), 2014 International Conference on*, pages 1295–1300, April 2014.

[11] W. K. Ma, T. H. Hsieh, and C. Y. Chi. Doa estimation of quasi-stationary signals via khatri-rao subspace. In *2009 IEEE International Conference on Acoustics, Speech and Signal Processing*, pages 2165–2168, April 2009.

[12] M. M. Goulding and J. S. Bird. Speech enhancement for mobile telephony. *IEEE Transactions on Vehicular Technology*, 39(4):316–326, Nov 1990.

[13] R. J. Talham. Noise correlation functions for

anisotropic noise fields. *The Journal of the Acoustical Society of America*, 69(1):213–215, 1981.

[14] M. Frikel,W. Utschick, and J. Nossek. Blind noise and channel estimation. In *Statistical Signal andArray Processing, 2000. Proceedings of the Tenth IEEE Workshop on*, pages 141–145, 2000.

[15] Shepard A. Clough and Francis X. Kneizys. Convolution algorithm for the lorentz function. *Appl.Opt.*, 18(13):2329–2333, Jul 1979.

[16] F. Li and R. J. Vaccaro. Performance degradation of doa estimators due to unknown noise fields. *IEEE Transactions on Signal Processing*, 40(3):686–690, Mar 1992.

[17] M. Bouri and S. Bourennane. High resolution methods based on rank revealing triangular factorizations. *International Journal of Mathematical,Computational,Physical, Electrical and Computer Engineering*, 1, 2007.

[18] S. W. Varade and K. D. Kulat. Robust algorithms for doa estimation and adaptive beamforming for smart antenna application. In *2009 Second International Conference on Emerging Trends in Engineering Technology*, pages 1195–1200, Dec 2009.

[19] Sylvie Marcos, Alain Marsal, and Messaoud Benidir. The propagator method for source bearing estimation. *Signal Processing*, 42(2): 121 – 138, 1995.

[20] V. T. Ermolaev and A. B. Gershman. Fast algorithm for minimum-norm direction-of-arrival estimation.*IEEE Transactions on Signal Processing*,42(9):2389–2394, Sep 1994.

[21] Youssef Khmou, Said Safi, and Miloud Frikel. Lorentzian operator for angular source localization with large array. *Journal of Telecommunications and Information Technology*, 4:98–105, 2015.