

Comparative Study of Blind Channel Equalization

Said Elkassimi¹, Said Safi¹, Bouzid Manaut²

¹Departament of Mathematic and Informatic Polydisciplinary Faculty Sultan Moulay Slimane University Beni Mellal, Morocco

²Laboratoire Interdisciplinaire de Recherche en Science et Technique (LIRST), Sultan Moulay Slimane University Beni Mellal, Morocco

Abstract: In this paper we present two algorithms for blind channel equalization. These algorithms are compared with the adaptive filter algorithms such as Constant Modulus Algorithm (CMA), Fractional Space CMA (FSCMA) and Sign Kurtosis Maximization Adaptive Algorithm (SKMAA). The simulation results in noisy environment and for different number of symbols show that the presented algorithms gives good results compared to CMA, FSCMA and SKMAA algorithms. The channel equalization is performed using the ZF and MMSE algorithms.

Keywords: Blind Equalization Algorithms, Filter Adaptive, CMA, FSCMA, SKMAA, 4QAM.

1. Introduction

Conventional equalization and carrier recovery algorithms for Minimizing Mean Square Error (MMSE) in digital communication systems generally require an initial training period during which a known data sequence is transmitted and properly synchronized at the receiver [1, 2]. On the other side the blind channel equalization allows to retrieve the unknown input sequence symbol to the remote channel without the use of training bits. In addition blind channel equalization has become an important topic in digital communications [3]. Blind methods use the received signal sequence and some a priori knowledge of the input sequence statistics [4]. Non minimum phase channel equalization was performed using high-order statistics methods [5-8] or other nonlinearities that are effective only with non-Gaussian distribution input sequences [9]. In this paper, we study the most popular adaptive blind equalization Bussgang algorithm (the Godard algorithm [10]) or constant modulus (CMA) [11] and Fraction Spaced CMA algorithms [10,12], thus another algorithm based on Kurtosis Maximization method called Sign Kurtosis Maximization Adaptive algorithm (SKMAA) [13]. In telecommunication, equalization means filtering the noise which is added to transmit signal through a channel. Conventional channel equalization uses an initial training phase, while blind channel equalization does not use this phase. According to the paper, there are some well know algorithm for blind channel equalization. These algorithms, like CMA, FS-CMA and SKMAA, are based on adaptive filter techniques. The paper proposes a new algorithm called Said1 which is based on ZF and MMSE. The result of this algorithm looks better when compared to CMA, FS-CMA and SKMAA.

2. Problem Statement

The symbol of blind equalization by adaptive filter algorithms is depicted in Fig. 1. Let us assume that the transmitted sequence $S(k)$ is the symbol sequence of independent distribution, or $S(k) \in A$. Where A is the windows character set. The input to the equalizer is defined by:

$$\begin{aligned} X(k) &= h(k).S(k) + V(k) \\ &= \sum_{i=1}^N h(i).S(k-i) + V(k) \end{aligned} \quad (1)$$

Or $h(k) = [h(1), h(2), \dots, h(k)]$ represents the channel parameter, $S(k)$ represent the input system and $V(k)$ is the additive colored Gaussian noise. The output of the equalizer can be defined by the following relationship

$$y(k) = f(k).X(k) \quad (2)$$

Where $f(k)$ is the tap weight vector and $X(k)$ is the input data vector of the equalizer at k^{th} instant sampling time.

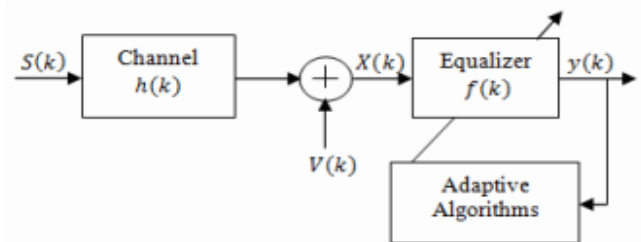


Figure 1. Blind Equalization by Adaptive Algorithms System Model

At sample k the output of the equalizer is defined by the relationship

$$y(k) = \sum_{i=1}^N f(i) \cdot X(k-i) \quad (3)$$

In this paper, we study the blind channel equalization algorithms, and we make a comparison between: the proposed algorithms and the filter adaptive algorithms such as the Constant Modulus Algorithm (CMA), Fractional-Spaced-CMA, and Sign Kurtosis Maximization Adaptive Algorithm (SKMAA).

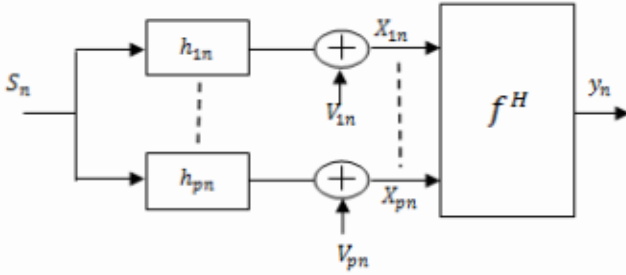


Figure 2. Equalization system model

3. Blind Channel Equalization

The aim of blind channel equalization is to retrieve the unknown input sequence symbol to the remote channel, i.e: calculate the symbol error rate (SER), in this case we used the adaptive filter algorithms.

3.1. ZF and MMSE Equalization

This algorithm is based on two traditional equalization algorithms such as: Zero Forcing (ZF) and Minimum Mean Square Error (MMSE) equalizations. So to developed this algorithm (algorithm1) we present first the ZF and MMSE algorithms.

3.1.1. ZF equalization

In order to compensate the effect of Power Line Communication (PLC) channel, we consider the ZF equalizer, which satisfies the condition shown below.

$$f_{ZF} H = I \quad (4)$$

Where $f_{ZF} = (H^H H)^{(-1)} H^H$ is the ZF decoding matrix, $(.)^H$ denotes the Hermitian transpose, H is the channel matrix and I is the identity matrix. Given the received signal X , the receiver can obtain the estimated signal by using the ZF equalization, which is given by:

$$\begin{aligned} \hat{S}(n) &= f_{ZF} X(n) \\ \hat{S}(n) &= f_{ZF} H S(n) + f_{ZF} V(n) \\ \hat{S}(n) &= I S(n) + f_{ZF} V(n) \end{aligned} \quad (5)$$

Where $\hat{S}(n)$ is an estimated matrix of the transmitted signal. If the determinant of H is not zero so that there exists the inverse matrix of H the decoding matrix can be expressed as

$$f_{ZF} = H^{-1} \quad (6)$$

The ZF equalization is ideal when the channel is noiseless. However, when the channel is noisy, the ZF equalization will amplify the noise greatly where the channel has small magnitude in the attempt to invert the channel completely.

3.1.2. MMSE equalization

In order to minimize the power of the noise component, we employ the MMSE equalization, which is given by:

$$f_{MMSE} = \min E \left\| S_{n-d} - f_{MMSE}^H X(n) \right\|^2 \quad (7)$$

Where f_{MMSE} is the MMSE decoding matrix and $\|.\|$ the norm of $S_{n-d} - f_{MMSE}^H X(n)$.

Or:

$$f_{MMSE} = R_X^{-1} \hat{h}_d \quad (8)$$

Where $R_X = E[X(n)X^H(n)]$ and $\hat{h}_d = E[X(n)S_{n-d}^*]$. In practice the expression of R_X and $\hat{h}_d$ is:

$$R_X = \frac{1}{k} \sum_{n=1}^k X(n)X^H(n) \quad (9)$$

$$R_X = \frac{1}{M} \sum_{n=1}^k X(n)S_{n-d}^* \quad (10)$$

The MMSE equalization can be used to estimate the channel effect with varying the decoding matrix in accordance with SNR. Besides, it prevents the noise component from being amplified.

3.2. Proposed equalizer (Algorithm 1)

The algorithm 1 equalizer is proposed for blind channel equalization is defined by the steps following:

- Transmit k symbols, the first M is assured known;
- Obtain received samples X_n ;
- Construct sample vectors $X(n)$;
- Calculate R_X and $\hat{h}_d$;
- Calculate $f_{MMSE} = R_X^{-1} \hat{h}_d$;
- Calculate symbol error rate (SER):
 - Use $f^H X(n)$ to estimate symbol $\hat{S}_{n-d}$;
 - Compare $\hat{S}_{n-d}$ with S_{n-d} .

3.3. Proposed equalizer (Algorithm 2)

Blind equalization is a process of recovering an unknown input data sequence from an observed noisy signal at the output of an unknown channel. The main advantage of blind channel equalization is that it does not require a training sequence, which would usually cause a reduction in the data rate. In this case we have proposed an algorithm to recover the symbols S . In

the absence of noise, the equalizer output is given by the following relationship:

$$y(n) = H^H G_i G_i^H X(n) \quad (11)$$

Where H represent the channel matrix, and G_i is the matrix calculated by using the SVD method. And the equalizer parameter is defined by the following equation:

$$f = H^{-1} \quad (12)$$

Then the estimated symbol is given by:

$$\begin{aligned} \hat{S} &= f^H X(n) \\ \hat{S} &= (H^H H)^{-1} H^H X(n) \end{aligned} \quad (13)$$

So, algorithm 2 equalizer is defined by the following steps:

- Construct sample vectors $X(n)$;
- Calculate R_X and f ;
- Calculate G_i by SVD method;
- Calculate symbol error rate (SER);
 - Use $f^H X(n)$ to estimate symbol $\hat{S}$;
 - Compare $\hat{S}$ with S .

3.4. CMA Equalizer

The problem with blind adaptive techniques is their poor convergence property compared to traditional techniques using training sequences. Generally a gradient descent based algorithm is used with the blind adaptation schemes. The most commonly used gradient descent based blind adaptation algorithm is the Constant Modulus Algorithm (CMA). The Constant Modulus Algorithm (CMA) [10,12] has gained widespread practical use as a blind adaptive equalization algorithm for digital communications systems operating over inter-symbol interference channels. The constant modulus (CM) criterion can be expressed by the cost function $J_{CM} = \frac{1}{4} E(|X_n|^2 - \gamma)^2$, where γ is a positive constant known as the Godard radius [1]. The equalizer update algorithm leading to a stochastic gradient descent of J_{CM} is known as the Constant Modulus Algorithm (CMA) and is specified by the following equation [15]:

$$f(n+1) = f(n) + \mu S^*(n) \underbrace{X_n(\gamma - |X_n|^2)} \quad (14)$$

Where μ is a step-size and $S^*(n)$ is the equalizer input vector at time index n . the asterisk denotes conjugation.

We note:

$$\psi(X(n)) = X_n(\gamma - |X_n|^2) \quad (15)$$

The function $\psi(X(n))$ identified in (15) is referred to as the CMA error function.

3.5. Fractional Spaced CMA Equalizer

For a Fractionally Spaced Equalizer (FSE), the tap spacing of the equalizer is a fraction of the baud spacing (in time) or the transmitted symbol period. As the output of the equalizer has the same rate as the input symbol rate, the output of the FSE needs to be calculated once in every symbol period [20]. FSCMA is used to directly estimate the equalizer f . It is similar to CMA [21]. In fractional space it is global convergences.

$$\min J = E[|f^H X(n)|^2 - R_z]^2 \quad (16)$$

Update rule:

$$f_{n+1} = f_n - \mu \cdot E[|f^H X(n)|^2 - R_z] X(n) X^H(n) f_n \quad (17)$$

Algorithm:

- Construct the received sample space;
- Construct the sample vector $X(n)$;
- For $n = 1, 2, \dots$
 - Update function (17)
 - Calculate instant error
- Check SER.

3.5. SKMAA Equalizer

Sign Kurtosis Maximization Adaptive Algorithm (SKMAA) is used to blind channel equalization. SKMAA is developed based on kurtosis of stochastic signals, the stochastic ascend approach, and sign algorithm for restoring the blind equalizer weight vectors [17]. Furthermore it is also necessary to calculate the output sample $y(n)$ of the equalizer filter once per algorithm update. The general idea is to maximize the chosen cost function. To assure that the SKMAA algorithm ascends for each time step n , the following requirement is made

$$J(f(n+1)) > J(f(n)) \quad (18)$$

Or the cost function is given by

$$J_{KMA}(f) = \frac{|Kurtosis[y(n)]|}{E^2[y(n)]^2} \quad (19)$$

Where $y(n)$ the equalizer is output and $Kurtosis[y(n)]$ is defined by:

$$Kurtosis[y(n)] = E y^4(n) - 3 E^2 y^2(n) \quad (20)$$

The update is then given by

$$\begin{aligned} f(n+1) &= f(n) + \mu \Delta f J_{KMA}(f) \\ &= f(n) + \mu F(y) [S(n) * h(n)] \\ &= f(n) + \mu F(y) X(n) \end{aligned} \quad (21)$$

Or

$$\Delta f J_{KMA}(f) = F(y) X(n) \quad (22)$$

Where F is the feedback function defined by:

$$F(y) = \frac{4(E[y^2]y^2 - E[y^4])y}{E^3[y^2]} \quad (23)$$

A Sign Algorithm is introduced into KMAA. So the SKMAA is written by:

$$f(n+1) = f(n) + \alpha \text{Sign}[F(y)] \cdot [X(n)] \quad (24)$$

Where $\text{Sign}[\cdot]$ is a simple sign is function, and α is a forgetting factor which is used to replace the ensemble averages by empirical averages which are then adaptively updated ($0 < \alpha < 1$)[17].

4. Simulation Results

4.1. Channel Equalization by Proposed algorithms and Adaptive Filter algorithms.

In this part we test the performance of proposed algorithms (Algorithm 1, Algorithm 2), CMA, Fractional-Space-CMA and SKMAA algorithms in order to equalize the output of the channel.

4.1.1 Performance of the adaptive filter equalizer CMA, FSCMA and SKMAA

In this part, we compare between the CMA, FSCMA and SKMAA equalizer that are being studied, for testing the performance. Tables I and II represents the Symbol Error Rate (SER) for different SNR values and number of symbols M.

Table1. Comparison the symbol error rate (SER) of filter adaptive algorithms for equalizer channel with different SNR values.

SNR	Equalizer	SER
0dB	CMA	0.4628
	FSCMA	0.4373
	SKMAA	0.3127
5dB	CMA	0.2789
	FSCMA	0.2554
	SKMAA	0.2278
10dB	CMA	0.1881
	FSCMA	0.1793
	SKMAA	0.2061
15dB	CMA	0.0369
	FSCMA	0.0337
	SKMAA	0.1745
20dB	CMA	0.0070
	FSCMA	0
	SKMAA	0.1544
25dB	CMA	0.0040
	FSCMA	0
	SKMAA	0.1474

Table2. Comparison the symbol error rate (SER) of filter adaptive algorithms for equalizer channel with different number of symbols.

Number of Symbols M	Equalizer	SER Value
512	CMA	0.4076
	FSCMA	0.3659
	SKMAA	0.3150
1024	CMA	0.4772
	FSCMA	0.3944
	SKMAA	0.3197
2048	CMA	0.5059
	FSCMA	0.4792
	SKMAA	0.3481
4096	CMA	0.5563
	FSCMA	0.5177
	SKMAA	0.3658

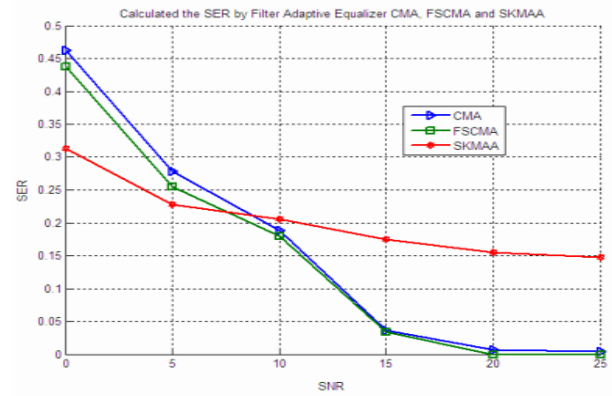


Figure 3. The SER in function of SNR by CMA, FSCMA and SKMAA Equalizer.

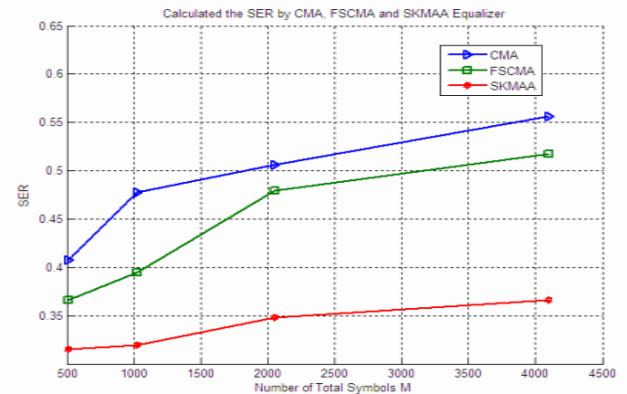


Figure 4. The SER in function of the number of symbols M by CMA, FSCMA and SKMAA Equalizer.

From the figure (Figure 3) we remark that the SKMAA equalizer is more performing in noisy environment ($\text{SNR} < 10\text{dB}$) than CMA and FSCMA algorithms. But if $\text{SNR} > 10\text{dB}$ we remark that CMA and FSCMA algorithms are more performing than SKMAA algorithm. In addition SKMAA equalizer is better than CMA and FSCMA equalizers, if the number of symbols M is larger (Figure 4).

4.1.2 Performance of the proposed algorithms versus Adaptive Filter Equalizer

this part we test the performance of proposed algorithms (Algorithm 1, Algorithm 2), and adaptive filter equalizer (CMA, FSCMA and SKMAA) for blind channel equalizer with different SNR values, total number of symbol, and we use the modulation QPSK or 4 QAM symbol sequence.

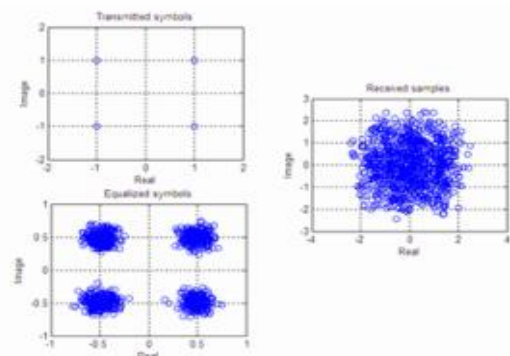


Figure 5. Channel equalization by proposed algorithm (Algorithm 1), with T=1024 and SNR=25 dB.

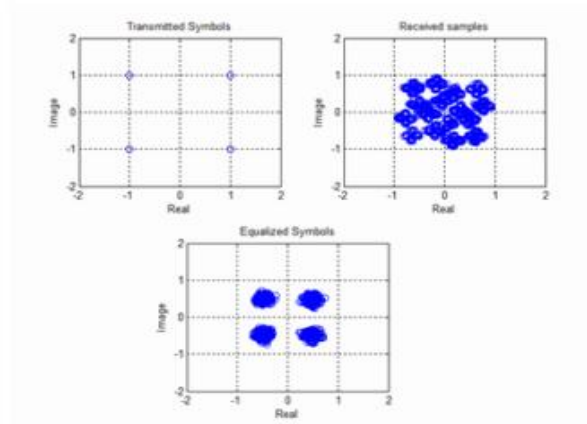


Figure 6. Channel equalization by proposed algorithm (Algorithm 2), with $T=1024$ and $SNR=25$ dB.

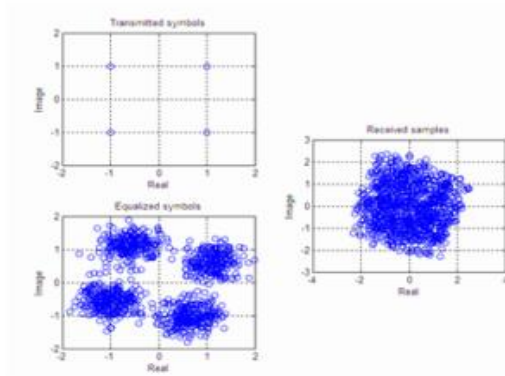


Figure 7. Channel equalization by CMA algorithm, with $T=1024$ and $SNR=25$ dB.

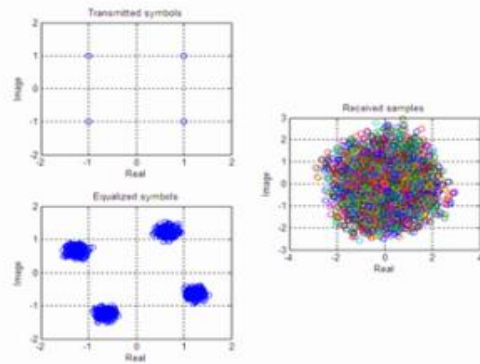


Figure 8. Channel equalization by Fractional-Space-CMA algorithm with $T=1024$ and $SNR=25$ dB.

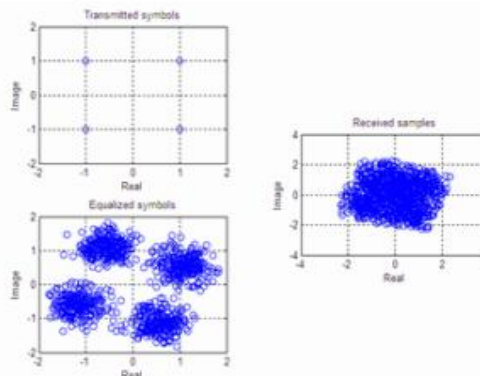


Figure 9. Channel equalization by SKMAA algorithm with $T=1024$ and $SNR=25$ dB.

From the simulation results, we notice that the proposed algorithms gives good channel equalization compared to filter adaptive equalizers(CMA, FSCMA and SKMAA). Thus the FS-CMA equalizer is better versus the CMA and SKMAA equalizers.

Table3. Comparison the symbol error rate (SER) of Proposed Algorithms and filter adaptive algorithms for equalizer channel with different SNR values.

SNR	Equalizer	SER
0dB	CMA	0.4628
	FSCMA	0.4373
	SKMAA	0.3127
	Algorithm 1	0.1564
	Algorithm 2	0.5722
5dB	CMA	0.2789
	FSCMA	0.2554
	SKMAA	0.2278
	Algorithm 1	0.1026
	Algorithm 2	0.4529
10dB	CMA	0.1881
	FSCMA	0.1793
	SKMAA	0.2061
	Algorithm 1	0.0817
	Algorithm 2	0.3536
15dB	CMA	0.0369
	FSCMA	0.0337
	SKMAA	0.1745
	Algorithm 1	0.0518
	Algorithm 2	0.1305
20dB	CMA	0.0070
	FSCMA	0
	SKMAA	0.1544
	Algorithm 1	0.0020
	Algorithm 2	0.0210
25dB	CMA	0.0040
	FSCMA	0
	SKMAA	0.1474
	Algorithm 1	0
	Algorithm 2	0.0008

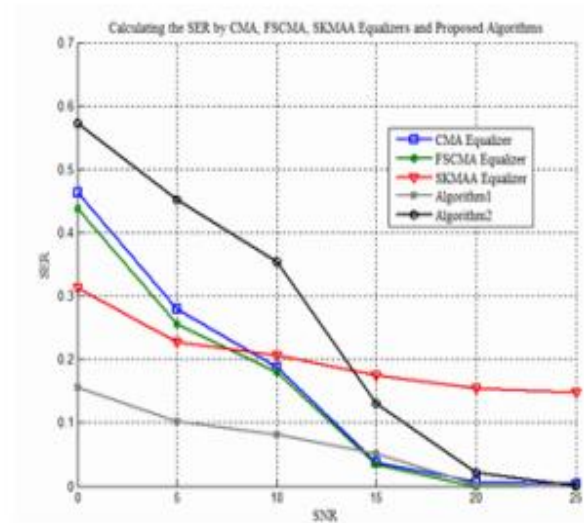


Figure 10. Calculating the SER in function the SNR values by CMA, Fractional-Space-CMA, SKMAA equalizers and proposed algorithms (Algorithm 1, Algorithm 2).

Table4. Comparison the symbol error rate (SER) of Proposed Algorithms and filter adaptive algorithms for equalizer channel with different number of symbols.

Number of Symbols M	Equalizer	SER Value
512	CMA	0.4076
	FSCMA	0.3659
	SKMAA	0.3150
	Algorithm 1	0.1992
	Algorithm 2	0.1398
1024	CMA	0.4772
	FSCMA	0.3944
	SKMAA	0.3197
	Algorithm 1	0.2331
	Algorithm 2	0.0463
2048	CMA	0.5059
	FSCMA	0.4792
	SKMAA	0.3481
	Algorithm 1	0.2494
	Algorithm 2	0.0265
4096	CMA	0.5563
	FSCMA	0.5177
	SKMAA	0.3658
	Algorithm 1	0.2812
	Algorithm 2	0.0022

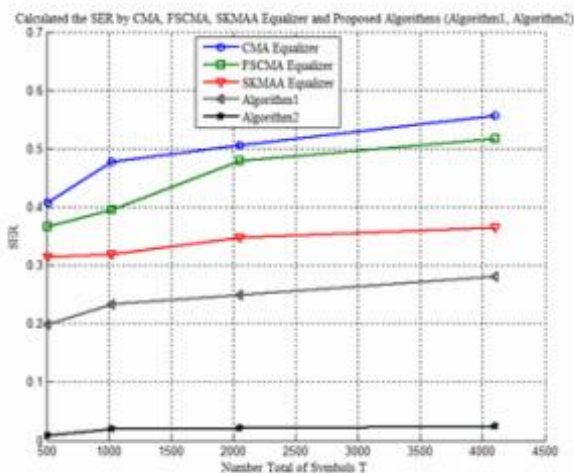


Figure 11. Calculating the SER in function the number Total of symbols T by CMA, Fractional-Space-CMA, SKMAA equalizer and proposed algorithms.

From the simulation results obtained, we remark that the Proposed Algorithms (Algorithm 1, Algorithm 2) gives a good channel equalization compared to CMA, FSCMA and SKMAA equalizers, moreover the algorithm 2 is better than algorithm 1 if the total number of symbols T is very high (Figure 10) or algorithm 1 is more performance than algorithm 2 in the environment noisy case (Figure 11).

5. Conclusions

In this paper we have presented two algorithms to solving the problem of blind channel equalization, and we have compared it with the adaptive filter equalizers. The simulation results show that the proposed algorithms are more performant compared to adaptive filter equalizers (CMA, FMCSA and SKMAA). In addition the proposed algorithm 2 is better than the algorithm 1 to blindly channel equalization if the total number of symbol is very high, or algorithm 1 is more

performance equalize channel than the algorithm 2 in noisy environment.

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research interests include communications, wide-band wireless communication systems, signal processing and system identification.



Universrrity, in 1997 and 2002, respectively. He has been a Professor of information theory and telecommunication systems at the National School for applied Sciences, Tangier, Morocco, from 2003 to 2005. Since 2006, he is a Professor of applied mathematics and programming at the Faculty of Science and Technics, Beni Mellal, Morocco. In 2008 he received the Ph.D. degree in Telecommunication and Informatics from the Cadi Ayyad University. His general interests span the areas of communications and signal processing, estimation, time-series analysis, and system identification – subjects on which he has published 14 journal papers and more than 60 conference papers. Current research topics focus on transmitter and receiver diversity techniques for single- and multi-user fading communication channels, and wide-band wireless communication systems.



2005, respectively. Professor of theoretical Physics from 2006 to now at the Polydisciplinary Faculty , Beni Mellal, Morocco. Ph.D. degree in Theoretical Physics, in 2011, from the Faculty of Science and Technics, USMS, Beni Mellal, Morocco. Current research topics focus on Theoretical Physics.

Said Elkassimi obtained the B.Sc. degree in Physics and M.Sc. degree in computer science from polydisciplinary faculty, in 2009 and from Faculty of Science and Technics Beni Mellal, Morocco, in 2012, respectively. Now he is Ph.D student and his

Said Safi received the B.Sc.degree in Physics (option Electronics) from Cadi Ayyad University, Marrakech, Morocco in 1995, M.Sc. and Doctorate degrees from Chouaib Doukkali University and Cadi Ayyad

Bouzid Manaut received the B.Sc.degree in Physics (Theoretical Physics) from Cadi Ayyad University, Marrakech, Morocco in 1999, M.Sc. and Doctorate degrees from Cadi Ayyad Universrry and Moulay Ismail University, in 2002 and