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A Customers' Discrete Choice Model for Competition over Incomes in Telecommunications Market

M'hamed Outanoute[1], Mohamed Baslam[1], Belaid Bouikhalene[2]
[1]Sultan MoulaySlimane University, Faculty of Sciences and Techniques, BeniMellal, Morocco
[2]Sultan MoulaySlimane University, Polydisciplinary Faculty, BeniMellal, Morocco
mhamed.outanoute@gmail.com, balsam.med@gmail.com, bbouikhalene@yahoo.fr

Abstract :*The customers churn between service providers (SPs) due to better prices, better qualities of services or better reputation. Each provider is assumed to look for a maximized revenue, which depends on the strategy of the competitor. Several works present in the literature were based on the price as the only decision parameter, whereas others parameters such as quality of service (QoS) have a decisive impact to register to an operator rather than the others. We formulate the interaction between SPs as a non-cooperative game. First, each SP chooses QoS to guarantee and the corresponding price. Second, each customer chooses his SP and may churn to another or alternatively switch to -no subscription state- depending on the observed price/QoS. In this work, we operate Markov chain to model user's decisions, which depend on the strategic actions of SPs. We adopt logit model to represent transition rates. Finally, we provide extensive numerical results to show the importance of taking price and QoS as joint decision parameters.*

Keywords :*Pricing, QoS, Migrating customers, Service providers' competition, Logit model.*

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1. Introduction

Theliberalizationoftelecommunicationsservicesandtheproliferationofserviceprovidershasledtorelevanceofcustomermigrationphenomenonbetweenoperatorsandtoaveryatrociouscompetitionintelecommunicationsmarket.Churnisespeciallylargein mobilenetworks,whereyearlymigrationratesashighas25%are notuncommon[15][22].Providersseekretaintheircustomersandattractnewones,countingonprevention[1][11]andonreactivestrategies[18][14].

Among factorsthatinfluencethecustomers'decisionstochangeoperator,priceplays animportantrole[15][8][9]. Today,withchangingtrafficcharacteristics,communicationsaremorestringentintheirqualityrequirements(deliveryrateandtime)thanrawdata.Hence,theneedofsomereconsiderationoftheimplicationsofarchitecture,serviceclasses,anddesignprinciples onthepricingmodelsofserviceproviders[7].Fromcustomers'pointofview,moretheguaranteedqualityofservicetendsto100%servicequality(fulldeliveryrateandreasonabledelay),theserviceproviderbecomesmoreattractive.Fromserviceproviderspointof view,generatesuchhighservicequalityreducestheirnetrevenue.

Now,itisclearthatpriceandservicequalitystimulateandinfluencethedecisionsofcustomerstochurnfromoneserviceprovider toanother.Serviceprovidersareseekingtoattractnewcustomers, butmostlyretaintheir existingcustomers. Thelossofonecustomerimpliesthelossoffuturerevenuesassociated tothat customer(a zero-sum game canmodelize suchsituations).This phenomenonis largelymodeled inseveral worksbasingonnon-cooperative gametheoryandassumingasinglecharacteristic,suchasprice[15][9][16][21],QoSintermsofdelay[19],or evenlossprobabilities[4].However,andtodevelopamodelsimilartoreality,other worksincorporatejointlypriceandsomemeasureofQualityofservice[7][10]. Formoredetailsonsurveystechniquesofcompetitive gameintelecommunications,see[3][5].

This work is organized as follows. In Section 2, we formulate a model of customers' behavior using a Markov chain and a description of probabilities to churn. Section 3 presents the description of the non-cooperative game between service providers and the strategies that define the best policy for both of them. In Section 4, we give a definition of the logit model used to expresses the rate of migration in terms of price and quality of service. In Section 5, we provide extensive numerical results to assist our analysis. Finally, in Section 6, we conclude our paper.

2. Costumers' Behavior Model

We consider a system with N customers and 2 service providers (SPs). Each SP i decides a QoS qi to guarantee and the corresponding price pi. To choose his operator, the customer seeks to find the SP that proposes a good compromise price/QoS. Then, each costumer decides to register to one of the two operators or to stay at no subscription state, see figure.1.

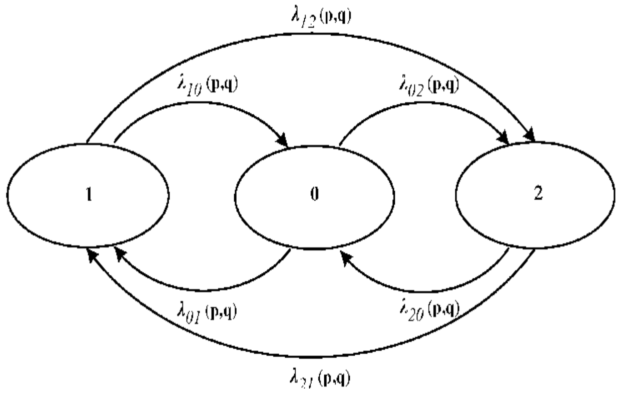

Figure 1: Customer's behaviour as a continuous time Markov chain.

From SP point of view, it shall to define the best pricing strategy and the best amount of bandwidth, to request from the network owner, for increasing his market share. From customer' point of view,the question is to set the best probabilities vector to register to an operator. The problem is thus conceived as aStackelberg game [2] where the operators are the leaders and the customers are the followers.

Using backward induction, the providers play first and anticipate the resulting strategy of end users. Each costumer may stay with his operator or leave it to register under services of another. We assume that the continuous time Markov chain that is depicted in figure 1 represents the behavior of a customer. Here state 1 means that the customer is with provider 1, state 2 that he is with provider 2 and state 0 that he does not use any service.

The transition from state *i* to state j depends not only on parameters of current SP *i* that are p_i and q_i, but also on the price and QoS offered by its competitors, that is, π_i depends upon the entire price vector p = (p_1, p_2) and the entire QoS vector q = (q_1, q_2). To avoid negative profits, we consider here that both price and QoS indicator are bounded and positives, i.e., $0<p_{min} \leq p_i \leq p_{max}$ and $0 < q_{min} \leq q_i \leq q_{max}$.

Let us denote by λ_{ij} the transition probability from state *i* to state *j*, the transition probabilities are depicted in figure 1. The resulting infinitesimal generator of Markov chain corresponding to the figure 1 is the following

Let $\pi(p,q) = \{\pi_i(p,q), i \in \{0,1,2\}\}$ denotes the steady state of the Markovian system, where the probability that a given customer is with service provider i is π_i. From non-cooperative game theory perspective, π is theequilibrium mixed strategy of any given customer. Equivalently, π_i can also be seen as the market share of SP i, itfollows that the average number of customers that are with SP i is $N\pi_i$. The customers' problem is then a solution ofthe system:

$$\begin{cases} \pi Q = 0, \\ \sum_{i=0}^{2} \pi_i = 1, \\ \pi_i \geq 0, i = 0,1,2. \end{cases} \qquad (1)$$

For two SPs, the solution of this system is given by:

$$\pi_0 = (\lambda_{10}\lambda_{20} + \lambda_{10}\lambda_{21} + \lambda_{12}\lambda_{20}) \div C,$$
$$\pi_1 = (\lambda_{01}\lambda_{21} + \lambda_{21}\lambda_{02} + \lambda_{20}\lambda_{01}) \div C,$$
$$\pi_2 = (\lambda_{02}\lambda_{12} + \lambda_{10}\lambda_{02} + \lambda_{01}\lambda_{12}) \div C,$$

Where:

$$C = \lambda_{10}\lambda_{12} + \lambda_{21}\lambda_{02} + \lambda_{02}\lambda_{10} + \lambda_{02}\lambda_{12} + \lambda_{20}\lambda_{01} + \lambda_{20}\lambda_{10} + \lambda_{20}\lambda_{12} + \lambda_{21}\lambda_{01} + \lambda_{21}\lambda_{10}.$$

3. Service Providers Strategic Decision

After modeling the behavior of customers, with taking into consideration both the unit price and the guaranteed QoS, we turn now to define the best policy (price and QoS) for each SP. We first define the utility function and then analyze the equilibrium concept. As it is known, each SP seeks to attract the maximum possible number of customers among a population ofN customers. On one hand, increasing the price (and/or decreasing the quality to guarantee) will increase the revenue per customer. On the other hand, this policy may potentially reduce the number of customers (equivalently the market share). Henceforth, there is a tradeoff to be analyzed.

Having a market share π_i, the total revenue of SP i is then N π_i. We assume that we have a single network owner, this latter charges each SP i a cost ϑ_i per unit of requested bandwidth. In order to insure the customers loyalty, the amount of bandwidth μ_i required by SP i should depend on π_i, N and on the QoS q_i it wishes to offer to its customers. Therefore, the net profit of SP i is simply the difference between the total revenue and the fee paid to the network owner:

$$U_i(p,q) = N\pi_i p_i - F_i(q_i, \pi_i), \forall i \in \{1,2\}. \qquad (2)$$

The fee paid by SP i can be written as [10]:

$$F_i = \vartheta_i \mu_i(N, \pi_i, q_i)$$

where $\mu_i(N, \pi_i, q_i)$ istheamountofbandwidthrequiredbySP *i*toguaranteethe announcedQoSq_i,whichhasthefollowingform:

$$\mu_i(N, \pi_i, q_i) = N\pi_i g_i(q_i) + h_i(q_i) \qquad (3)$$

where g_i *(q_i)* and h_i *(q_i)* are positive functions, which mean that the profile function of SP *i* becomes:

$$U_i(p,q) = N\pi_i(p_i - \vartheta_i g_i(q_i)) - \vartheta_i h_i(q_i), \forall i \in \{1,2\}. \qquad (4)$$

Expected Delay as QoSIndicator: In the rest of this paper, we assume that the measure defining the QoS corresponds to some function of the expected delay. We consider the Kleinrock delay function, which is a common delay function used in networking games, see [10] and [19]. In this way, the maximization of QoS requires minimization of the delay. For this reason, and instead of minimizing delays, we consider the maximization of the reciprocal of its square root:

$$q_i = 1/\sqrt{\mathrm{Delay}_i} = \sqrt{\mu_i - N\pi_i}. \quad (5)$$

Therefore, $\mu_i(N, \pi_i, q_i) = N\pi_i g_i(q_i) + h_i(q_i)$

We will focus on the rest of this paper to the simple case where $g_i(q_i) = 1$ and $h_i(q_i) = q_i^2$.

Thus,equation(4)becomes:

$$U_i(p, q) = N\pi_i(p_i - \vartheta_i) - \vartheta_i q_i^2, \forall i \in \{1, 2\}. \quad (6)$$

Each service provider strives to find its best strategy, i.e., its price and QoS to guarantee maximizing its revenue, which can be modified by the strategy of the competitor. The solution concept to adopt here is naturally that ofa Nash equilibrium.

Definition1(NashEquilibrium): SincethecustomerssteadystatedependsonlyonthefixedpricesandtheofferedsQoSbythetwoadversarialSP,then $(p^*, q^*) = (p_1^*, p_2^*, q_1^*, q_2^*)$ isaserviceprovidersNashEquilibriumifit satisfies:

a) $(p_i^*, q_i^*) \in \underset{p_i, q_i}{\mathrm{argmax}}\, U_i(p_i, p_{-i}^*, q_i, q_{-i}^*), i = 1, 2$ and

b) (p^*, q^*) is a feasible strategy profile.

Algorithm1: Numerically finding the Nash equilibrium of the joint Price-QoS game.

a) Input:TransitionratesoftheMarkovchainasfunctionof*p*and *q*,

b) For each SP i

Forallfeasiblecoupleofvalues (p_i, q_i) findtheset $BR_{-i}(p_i, q_i)$,

c) Each 4-tuple $(p_1^*, p_2^*, q_1^*, q_2^*)$ suchthat $(p_1^*, q_1^*) \in BR_1(p_2^*, q_2^*)$ and $(p_2^*, q_2^*) \in BR_2(p_1^*, q_1^*)$, isaNash equilibrium.

In order to solve this non-cooperative game we use a backward induction technique. We start with thecustomers' registration and derive their steady behaviour as a function of the price set and the QoS offered by the service providers. Without extra assumptions, existence of a Nash equilibrium cannot be ensured, nor its uniqueness when existence is shown. In the case where utilities and rates functions are simple enough in terms of prices and QoS, we may find the form of the Nash equilibrium analytically. Otherwise, the computations can be performed numerically using the following algorithm 1. We define the best response of each provider as a function of the strategy of itsopponent by:

$$BR_1(p_2, q_2) = \underset{p_1, q_1}{\mathrm{argmax}}\, U_1(p, q), \quad (7)$$

$$BR_2(p_1, q_1) = \underset{p_2, q_2}{\mathrm{argmax}}\, U_1(p, q). \quad (8)$$

4. Migration Rate Model

The transition rate describing the customer migration probabilities from one operator to another, depend on both prices and QoSs defined by providers. The choice of simplified cases has shown that the Nash equilibrium can be obtained analytically, but not easily [15][7]. In this section, we are interested in using a model as close as possible to reality, which expresses the rate of migration in terms of prices and QoSs.

4.1. ChurnRatesintheLiterature

Several studies have been conducted to identify churn determinants, which are the most relevant factors in determining churn. These works are based on both parametric and non-parametric approaches, for quantitative expression of churn rates in terms of prices and QoSs. We cite, among the nonparametric approaches, the use of neural networks and decision trees [17], and a learning evolutionary algorithm [6]. In this work, we focus on parametric approaches to obtain a closed form relationship. The logit model is the most widespread adopted in the literature to represent this relationship. This model employs a logistic probability distribution function [12][13][20]. This function is a linear function of a number of determinants churn. The probability that a user churns in the next period is given by the expression:

$$P_{churn} = \frac{1}{1 + e^{-I}}, \quad (9)$$

where I is the logit factor, given by

$$I = \sum_{i=1}^{n} \beta_i X_i, \quad (10)$$

where $X_i, i = 1, \ldots, n$ arethechurndeterminantsand $\beta_i, i = 1, \ldots, n$ are the coefficients representing therelativeimportanceofthosedeterminants.

4.2. Transition Rates

Inthispaperwehavefocussedonthepriceand QoS,sothatwecangrouptheimpactoftheotherchurndeterminantsintheoverallterm γ,arrivingatthesimplerexpression:

$$P_{churn} = \frac{1}{1+\gamma e^{-(\beta_1 p_i/p_j+\beta_2 q_j/q_i)}}, \qquad (11)$$

for the probability that in the specified period the user switches from state *i* to state *j*. We may employ that expression for a time period of any duration, so we can adopt it in the Markov chain model described in Section 2. We note that, according to expression (11), there is a non zero probability, namely $P_{churn} = \frac{1}{1+\gamma}$, that the user switches provider due to the ensemble of other dissatisfaction factors, even when the service offered by the losing provider is free and/or its QoS is very high.

The literature considers discretized time, whereas we focus here on a continuous time model. That latter difficulty is addressed here by assuming that timeperiods considered in Subsection 4.1, are short with respect to the mean sojourn time in a given state. This implies that the discrete time transition probabilities are approximately the continuous-time transition rates multiplied by the period duration. Consequently, we would like to consider transition rates from state $i \in \{0,1,2\}$ to state $j \in \{0,1,2\}$ of the form:

$$P_{churn} = \frac{k}{1+\gamma_i e^{-(\beta_1 p_i/p_j+\beta_2 q_j/q_i)}}, \qquad (12)$$

where $k > 0$ representstheinverseoftheperiodduration[15]. Since β_1 and β_2 representrespectivelytheusersensitivitytopricesandtheusersensitivitytoQoS,weconsideritisthesameforthedifferentstatesofthemodel.Weintroduceasymmetryamongprovidersthroughtheparameter γ_i asexplainedbefore,thisparameterencompassesthereasonsotherthanpriceandQoS(e.g.,reputation ...),whyausershouldleavestate*i*.

Themodelparametersthatweconsiderarethen:

- theusersensitivitytopriceandQoS β_1 and β_2,
- thelikeliness γ_i tostayincurrentstate $i, i \in \{0,1,2\}$,
- theinverse of period duration*k*(this parameter should not playa role in our model,since by a time unitchangewecanassume*k*=1),
- theuserperceivedcost$p_0$fornotbenefittingfromtheserviceand$q_0$thesupposedQoSinnon-subscription state.

5. Numerical Results and Discussions

Considering the previous expressions of the transition rates, we suggest to study the game proposed in this work. The dependence of those rates on provider prices and QoS are too complicated to solve the problem analytically, therefore we perform here a numerical study. The parameter values considered in this section are the following: N=10000,p_0=1,q_0=1,β_1=0.5,β_2=0.3,k=1, γ_0=3,γ_1=1,γ_2=2.

5.1. Price Game with Fixed QoS

Inthissubsection,wewillanalyzethebehaviorofthesystemifthequalityofserviceisfixedandthepriceisconsideredtheonlyactionofthegame.Expression(12)wouldimplythatalltransitionratesbeinaninterval $[k/(1+\gamma_i),k]$ regardlessofthepriceandQoSvalues.InthisgamewithfixedQoS,thisisnotrealistic,sinceitwouldimplythataprovidercouldensureanarbitrarilylargerevenuebysettingaverylargeprice. Wethereforeneedthatthetransitionratestoprovider***i***tendto 0and/orthattheratesfromprovider***i***tendto ∞,where p_i tendto ∞ and/or q_i tendto0.

Tothatend,weslightlymodifythepreviousexpression,andtaketransitionratesoftheform:

$$\lambda_{ij}(p_i,p_j,q_i,q_j) = \frac{k}{\gamma_i e^{-(\beta_1 p_i/p_j+\beta_2 q_j/q_i)}} = \frac{k}{\gamma_i} e^{\beta_1 p_i/p_j} e^{\beta_2 q_j/q_i} \qquad (13)$$

Figure 2 plots the utility U_1 of provider 1 when its price p_1 varies, for different values of the adversary price p_2. We remark that the net revenue of provider 1 is first increasing, and then decreasing in p_1. Also that the utility of provider 1 decreases when its price p_1 tends to infinity, which implies existence of a finite price p_0 maximizing U_1. This price value constitutes the best response of provider 1 against the price set by provider 2. We remark also that the net revenue of SP 1 increase when adversary price p_2 increase, which is likely intuitive. Indeed, if the opponentchooses a higher pricing strategy, then it may lose a fraction of its customers. Moreover, service provider SP 1 may attract those latter and make incite them to churn. Consequently, its net revenue is increased. Surprisingly, we observed an interesting feature characterizing our proposed price game: Through hundreds of numerical example runs, we 1) checked existence of a unique best response, and 2) checked that as the price defined by SP 2 increases, the best response of SP 1 tends to increase also! This motivated us to work about proving super-modularity of this game.

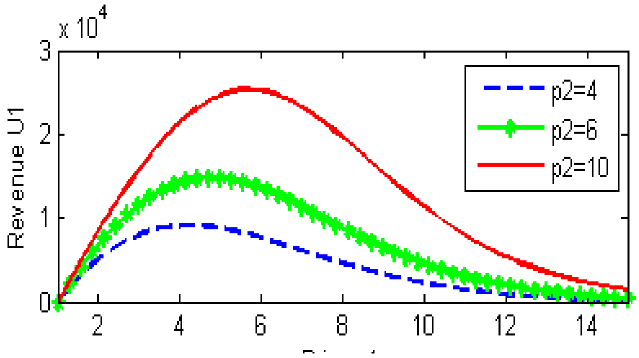

Figure 2 : Netrevenueofprovider1versus$p$1fordifferentvaluesof$p$2,with$q$1=15and q2=15.

Figure 3 and 4 present respectively the influence of QoS q1 and q2 of both providers on the net revenue U1 of SP 1. In terms of service provider's own quality of service, the net revenue behaves in two ways. When SP 1 sets a low price, it tends to be more beneficial to request relatively low amount of bandwidth providing low quality of service. This is a quite surprising behaviour, but can have a realistic good extent. Indeed, when an SP offers some given service at a low cost, its market share goes to 1. Absorbing the total number of customers, the tagged SP tends to no investing anymore and the offered service quality becomes poor. Another important result is that as the offered QOS increases as the best response increase respecting by the way: "If you want more, you have to pay more".

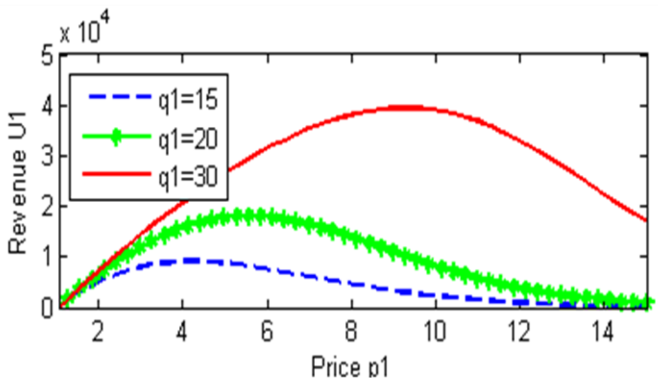

Figure 3 : Net revenue of provider 1 versus p1 for different values of q1, with q2 = 15 and p2 = 4.

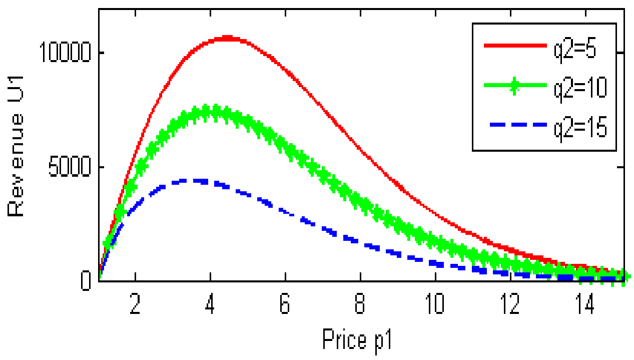

Figure 4 : Net revenue of provider 1 versus p_1 for different values of q_2, with q_1 = 10 and p_2 = 4.

Next, we depict the best response of the two adversarial service providers. We recall that this figure is obtained using algorithm 1 and the intersection between the two graphs represents the Nash equilibrium point of the game. Through several examples, we always obtained a unique Nash equilibrium. Another visible feature is that the fixed price should evolve in the same direction of the investment. In other words, the end price increases as the offered QoS increases.

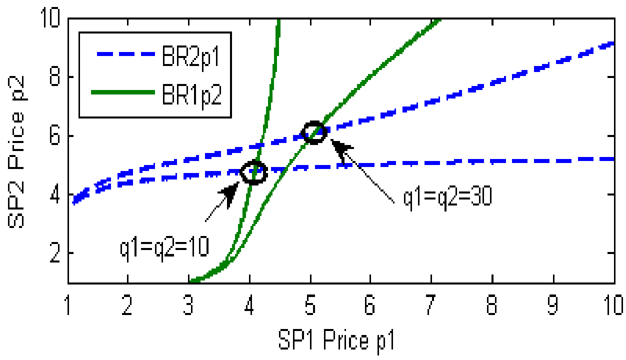

Figure 5 : Best response price for both providers for different values of $q_1 = q_2$.

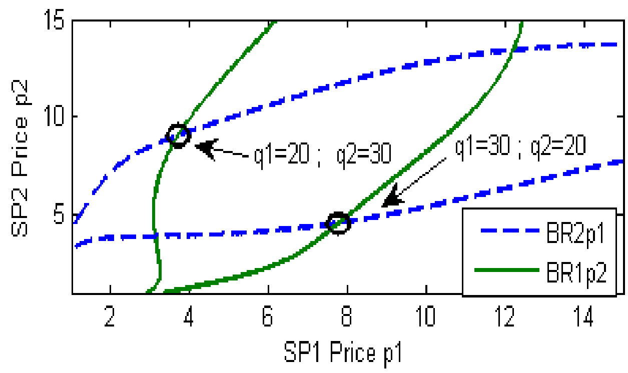

Figure 6 : Best responses price for both providers for different values of $q_1 \neq q_2$.

5.2. QoS Game with Fixed Price

In this subsection, we will analyze the behavior of the system if the price is fixed and the quality of service is considered the only action of the game. We first notice that the net revenue is decreasing with the QoS. This is quite intuitive since it means for service provider to invest more and more while charging customers a constant price, see figure 7. It is clear from the same figure that, as the opponent SP 2 increases its quality of service, it is beneficial for SP 1 to increase its QoS. This latter result is well known in economics.

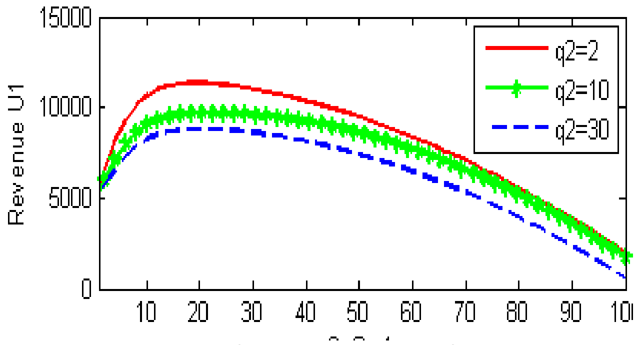

Figure 7 : Net revenue of provider 1 versus q_1 for several values of q_2, with p_1 = 4 and p_2 = 4.

Figure 8 (respectively 9) shows that the utility function of SP 1 is concave w.r.t q_1 for different prices p_1 (respectively p_2). This means that the best response in terms of QoS is unique and therefore the existence of the equilibrium in term of QoS is assured. On the other hand, the same figures show the need to define a higher price as the own QoS improves and the need investment (improve promised QoS) as the competitor reduces its price.

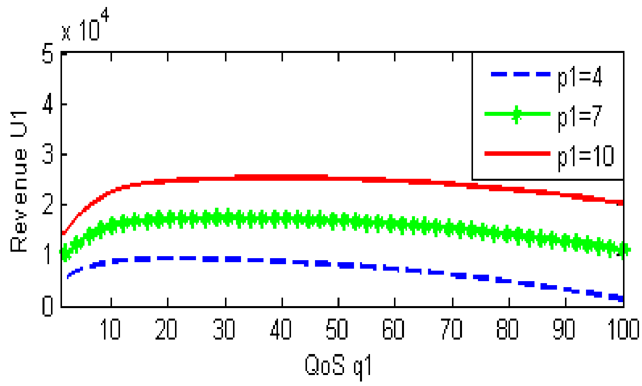

Figure 8 : Net revenue of provider 1 versus q_1 for several values of p_1, with q_2 = 15 and p_2 = 4.

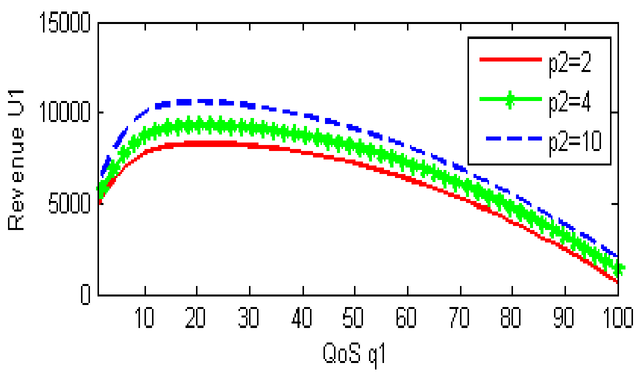

Figure 9 : Net revenue of provider 1 versus q_1 for several values of p_2, with q_2 = 15 and p_1 = 4.

Later, we plot the best response in terms of QoS while the two service providers set the same price (figure 10) and where they set adopt different pricing strategies (figure 11).

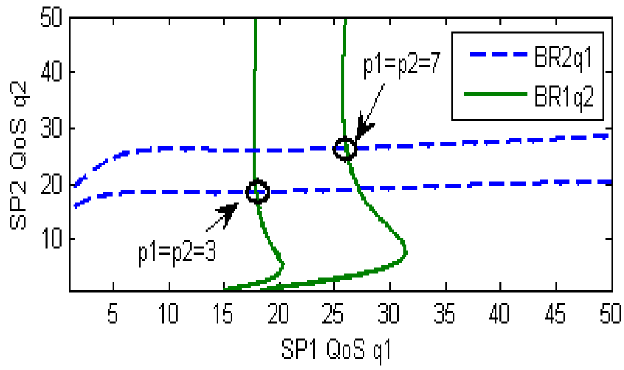

Figure 10 : Best response QoS for both providers for different values of $p_1 = p_2$.

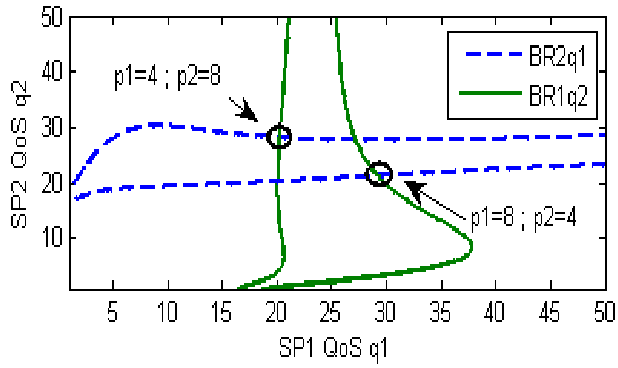

Figure 11 : Best responses QoS for both providers for different values of $p_1 \neq p_2$.

5.3. Convergence to Nash Equilibrium

Ifserviceprovidersallowfreecostservicewithpossiblynullquality,i.e.,priceandqualitymaybe0,wenoticethat

$$(p_1, p_2) = (0,0)$$

isalsoasatisfyingsituationforbothSPs,thusitrepresentsalsoanequilibriumpoint.Sinceitbringsanegativerevenuetotheproviders,andmoreoveritisnotastableNashequilibrium:ifanyofthetwoproviderss lightlydeviatesfromthatsituationbysettingastrictlypositiveprice,thenaniterativebestresponse-basedalgorithmleadstotheother(stable)Nashequilibrium.Byimposingnon-nullpriceandnon-nullquality,theNashequilibriumwouldbeaprioriuniqueandthesingle-parametergamewouldbesupermodular.Wewillconsequently focusonthatequilibriuminthefollowing,whenitexists.Algorithm2insuresconvergenceofbothSPstotheNashequilibrium pointshighlightedinpreviousfigures,seefigure12andfigure 13.Itisclearthatthespeedofconvergence isrelativelylow(around6iterationstoconvergetoprice-basedequilibriumandQoS-basedequilibrium).

Algorithm 2: ConvergencetoNashequilibrium(whenexists)

1: $z_i^{t+1} = BR_i(z_{-i}^t),\ i = \{1,2\}$.

where $BR_i(.)$ isthebestresponsefunctionofSPi,and z_i isagenericvariablerepresenting p_i or q_i (whilemaintainingtheotherparameterconstant).

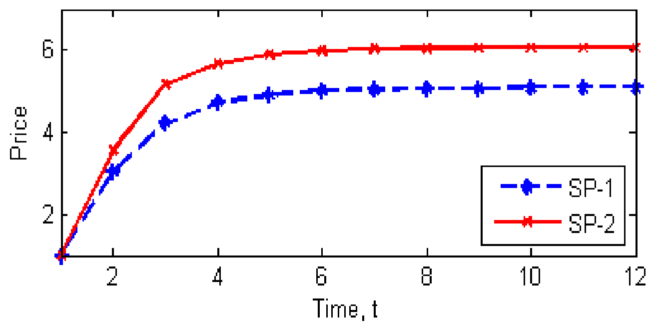

Figure 12: Pricing game: Convergence to NE under $(q_1, q_2) = (30, 30)$.

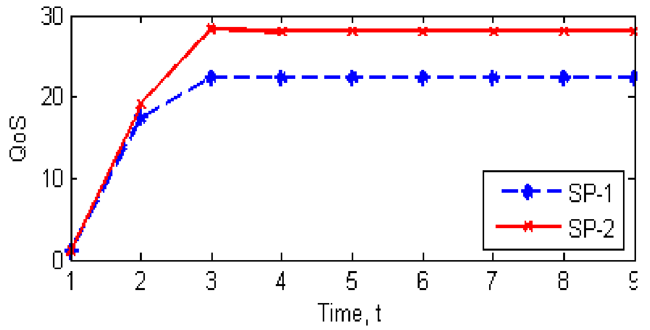

Figure 13: QoS game: Convergence to NE under $(p_1, p_2) = (5, 8)$.

6. Conclusion

In this work, we have presented a contribution to the study of the competition between telecommunications operators. These take into account the customer migration behavior to decide the price and quality of service to offer in the aim of maximizing their revenue. The problem is presented as a non-cooperative game based on the price and quality of service as joint decision parameters. The customer migration behavior is modeled by a Markov chain where the transition rates are expressed using the logit model. We have shown through numerical analysis the existence of many Nash equilibrium or none for this game, which can be unique if the game is symmetrical and the minimum price p_{min} and minimum quality of service q_{min} are not zero.

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