ACIT'2017

The International Arab Conference on Information Technology

Yassmine Hammamet, Tunisia

December 22-24, 2017

Applying an Algorithm on GSM Model Using PSK Constellations

[1]Jilali ANTARI, [2]Mustapha BOUKRI and [1]Abdessamad MALAOUI
[1]Polydisciplinary Faculty Taroudan, Morroco
[2]LMTI,Faculty of Sciences Agadir, Morroco

Abstract—In this paper, we apply an algorithm based on fourth order cumulants (ALG-CUM4) on GSM (Global System Mobile) model using PSK (Phase-Shift Keying) constellations. The goal of the application is the equalization of communication channel in GSM. The simulation results and the comparison with another algorithm, also based on fourth order cumulants using ISI (Inter-Symbol Interference) criteria and PSK constellations, demonstrates the performances of the algorithm (ALG-CUM4) in different SNR (signal-to-noise ratio).

Index Terms—Simulation; GSM model; Equalization; Fourth Order Cumulants.

I. Introduction

The identification parameters are very important and interesting to identify linear systems. They are used in different applications such as signal processing, blind equalization, time daily estimation and data communication. We have important results based on blind identification parameters type of finite impulse response (FIR) single-input single-output (SISO) communication channels from the output second-order statistics. In the literature, several studies were done without using any restrictive assumption on channel zeros, color of additive noise, channel order overestimation errors, and without increasing the transmission rate of the data stream on the one hand, and on the other hand, using the finite impulse response (FIR) system of identification based on cumulants techniques such as second, third and fourth order cumulants of system output [1]–[3].

Blind identification of linear time-invariant systems in particular MA single-input single-output systems has attracted much attention recently. Indeed, several approaches to identify the parameters of such systems were proposed in the literature [4]- [6]. Some of these are based on the second order cumulants (Autocorrelation Function ACF and power spectrum) of the observed sequences. But these approaches are sufficient only to identify the systems excited by Gaussian processes with minimum phase. However, in several applications, the observed signals are non-Gaussian and can be considered the output of a linear system excited by a non-Gaussian distribution input or a non-linear system excited by a Gaussian distribution input. Many **H**igh **O**rder **S**tatistics **(HOS)** based methods have been proposed to identify the systems excited by non-Gaussian processes with non-minimum phase [7], [8]. Another interesting property of **HOS** techniques is that they are insensitive to additive white or colored Gaussian measurement noise. These methods combine great value in many signal processing applications such as channel equalization in data communication, array processing and source separation, time delay estimation and image processing [9]–[11]. For this reason, in this paper we propose an algorithm based only on fourth order cumulants. The solution, in least squares sense, gives an estimation of linear parameters of GSM channel. The paper is organized as follows: sections II and III describe the issue of channel modeling and the different hypotheses used in the identification of communication systems. In section IV, we illustrate the expression of cumulants techniques used in the demonstration of the proposed algorithm (ALG-CUM4) presented in section V. An algorithm based on fourth order cumulants that will be used in the comparison with ALG-CUM4 is introduced in section VI. In sections VII and VIII, we present the equalization model compared to P. Comon algorithm [2], simulation and discussion using ISI, SNR and impulse response of global system in imaginary and real part representation. The results show the performance of ALG-CUM4 algorithm. Finally, conclusions are in section IX.

II. Model presentation

The output of a model (Fig. 1) that is excited by an unobservable input is described by the following formula:

$$y(k) = \sum_{i=0}^{q} h(i)e(k-i) \qquad (1)$$

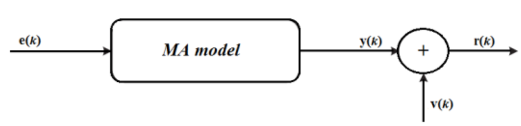

Fig. 1. Channel model.

where:

$$r(k) = y(k) + v(k) \qquad (2)$$

$\{e(k)\}$ is a non-Gaussian input sequence, $\{h(i)\}$ represents the impulse response coefficients, q is the order of the finite impulse response (FIR) system, $\{y(k)\}$ is the output of the system and $\{v(k)\}$ is the noise sequence.

III. INPUT HYPOTHESES $\{e(k)\}$

We use some hypotheses to simplify the parametric identification of communication systems. The hypotheses are as follows:

- H1: The input sequence $\{e(k)\}$ is independent and identically distributed **(i.i.d)** zero mean, with variance $\sigma_e^2 \approx 1$ and non-Gaussian;
- H2: The system is causal, i.e. $h(i) = 0$ for $i < 0$ and $i > q$, where $h(0)=1$;
- H3: The noise sequence $\{v(k)\}$ is assumed to be zero mean, i.i.d, Gaussian and independent of $\{e(k)\}$ with unknown variance.

IV. EXPRESSION OF CUMULANTS

We present the general fundamental relationships that allow to identify the channel model using Higher Order Cumulants (HOC).

The m^{th} order cumulants of the $\{y(k)\}$ can be expressed as a function of impulse response coefficients $\{h(i)\}$ as follows [12]:

$$\mathbf{C}_{m,y}(\tau_1,\ldots,\tau_{m-1}) = \mu_{m,e}\sum_{i=0}^{q} h(i)h(i+\tau_1)\ldots h(i+\tau_{m-1}) \tag{3}$$

with $\mu_{m,e}$ represents the m^{th} order cumulants of the excitation signal at origin.

If we take $m = 2$ into (3) we obtain the autocorrelation function (ACF):

$$\mathbf{C}_{2,y}(\tau) = \mu_{2,e}\sum_{i=0}^{q} h(i)h(i+\tau) \tag{4}$$

For $m = 4$ into equation (3), we obtain the fourth order cumulants:

$$\mathbf{C}_{4,y}(\tau_1,\tau_2,\tau_3) = \mu_{4,e}\sum_{i=0}^{q} h(i)h(i+\tau_1)h(i+\tau_2)h(i+\tau_3) \tag{5}$$

The Fourier transform of the 2^{nd} and 4^{rd} order cumulants are given respectively in the following equations:

$$\begin{aligned}\mathbf{S}_{2,y}(\omega) &= TF\{\mathbf{C}_{2,y}(\tau)\} \\ &= \mu_{2,e}\sum_{i=0}^{q}\sum_{tau=-\infty}^{+\infty} h(i)h(i+\tau)\exp(-j\omega i) \\ &= \mu_{2,e}H(-\omega)H(\omega)\end{aligned} \tag{6}$$

with $H(\omega) = \sum_{i=0}^{\infty} h(i)\exp(-j\omega i)$

$$\begin{aligned}\mathbf{S}_{4,y}(\omega_1,\omega_2,\omega_3) &= TF\{\mathbf{C}_{4,y}(\tau_1,\tau_2,\tau_3)\} \\ &= \mu_{4,e}H(-\omega_1-\omega_2-\omega_3)H(\omega_1)H(\omega_2)H(\omega_3)\end{aligned} \tag{7}$$

Using equations (6) and (7), we build a relationship between the spectra, the bispectra and the parameters of the output systems.

Therefore, if we take $\omega = \omega_1 + \omega_2 + \omega_3$, the equation (6) becomes:

$$\mathbf{S}_{2,y}(\omega_1+\omega_2+\omega_3) = \mu_{2,e}H(-\omega_1-\omega_2-\omega_3)H(\omega_1+\omega_2+\omega_3) \tag{8}$$

Then, from the Eqs. (7) and (8) we obtain the following relationship:

$$\mathbf{S}_{4,y}(\omega_1,\omega_2,\omega_3)H(\omega_1+\omega_2+\omega_3) = \mu_{4,2}H(\omega_1)H(\omega_2)H(\omega_3)\,\mathbf{S}_{2,y}(\omega_1+\omega_2+\omega_3) \tag{9}$$

with $\mu_{4,2} = \frac{\mu_{4,e}}{\mu_{2,e}}$.

The reversed Fourier transform of the Eq. (9) is:

$$\sum_{i=0}^{q}\mathbf{C}_{4,y}(\tau_1-i,\tau_2-i,\tau_3-i)h(i) = \mu_{4,2}\sum_{i=0}^{q} h(i)h(\tau_2-\tau_1+i)\,h(\tau_3-\tau_1+i)\mathbf{C}_{2,y}(\tau_1-i) \tag{10}$$

On the basis of the relationship (10), we develop the following algorithm in section V.

V. PROPOSED ALGORITHM: ALG-CUM4

If we take $\tau_1 = \tau_3$ into Eq. (10), we obtain:

$$\sum_{i=0}^{q}\mathbf{C}_{4,y}(\tau_1-i,\tau_2-i,\tau_1-i)h(i) = \mu_{4,2}\sum_{i=0}^{q} h^2(i)h(\tau_2-\tau_1+i)\,\mathbf{C}_{2,y}(\tau_1-i) \tag{11}$$

If we use the ACF property of the stationary process ($\mathbf{C}_{2,y} \neq 0$ only for $-q \le \tau \le q$ and vanishes elsewhere) and if we suppose that $\tau_1 = 2q$, the Eq. (11) becomes:

$$\sum_{i=0}^{q}\mathbf{C}_{4,y}(2q-i,\tau_2-i,2q-i)h(i) = \mu_{4,2}h^2(q)h(\tau_2-q)\mathbf{C}_{2,y}(q) \tag{12}$$

If the Eq (12) is causal (i.e. $h(i) = 0$ for $i < 0$ and $i > q$), the choice of τ_2 imposes that ($\tau_2 \ge q$). As a result, $0 \le \tau_2 - q \le q$. For this reason, we have $\tau_2 = q, q+1..., 2q$.

If we take $\tau_1 = \tau_2 = -q$ into the Eq (11), we obtain the following equation:

$$\sum_{i=0}^{q}\mathbf{C}_{4,y}(-q-i,-q-i,-q-i)h(i) = \mu_{4,2}\sum_{i=0}^{q} h^3(i)\mathbf{C}_{2,y}(-q-i) \tag{13}$$

According to the ACF property, the relation (13) is valid if $i = 0$. From Eq (13), we obtain:

$$\mathbf{C}_{4,y}(-q,-q,-q)h(0) = \mu_{4,2}h^3(0)\mathbf{C}_{2,y}(-q) \tag{14}$$

with $h(0) = 1$:

$$\mathbf{C}_{4,y}(-q,-q,-q) = \mu_{4,2}\mathbf{C}_{2,y}(-q) \tag{15}$$

Using the property of the cumulants:

$$\mathbf{C}_{4,y}(\tau_1,\tau_2,\tau_3) = \mathbf{C}_{4,y}(-\tau_1,\tau_2-\tau_1,\tau_3-\tau_1)$$

The equation (15) becomes:

$$\mathbf{C}_{4,y}(q,0,0)=\mu_{4,2}\mathbf{C}_{2,y}(q) \tag{16}$$

We use (Eq.16) to eliminate $\mathbf{C}_{2,y}(q)$ in Eq. (12). From Eq. (16), we obtain:

$$\sum_{i=0}^{q}\mathbf{C}_{4,y}(2q-i,\tau_2-i,2q-i)h(i)=h^2(q)h(\tau_2-q)\mathbf{C}_{4,y}(q,0,0) \tag{17}$$

To simplify Eq. (17), we consider equation (5) with $\tau_1=\tau_2=q$ and $\tau_1=\tau_2=q$, $\tau_3=0$, and then we obtain two relationships:

$$\mathbf{C}_{4,y}(q,q,\tau_3)=\mu_{4,e}h^2(q)h(\tau_3) \tag{18}$$

$$\mathbf{C}_{4,y}(q,q,0)=\mu_{4,e}h^2(q) \tag{19}$$

Using Eqs. (18), (19) and $\tau_3=q$, we obtain:

$$h(q)=\frac{\mathbf{C}_{4,y}(q,q,q)}{\mathbf{C}_{4,y}(q,q,0)} \tag{20}$$

From Eq. (20), we get the following:

$$h^2(q)\mathbf{C}_{4,y}(q,0,0)=\left(\frac{\mathbf{C}_{4,y}(q,q,q)}{\mathbf{C}_{4,y}(q,q,0)}\right)^2\mathbf{C}_{4,y}(q,0,0)=\alpha \tag{21}$$

If we use Eqs (21) and (17), we get the proposed algorithm based only on the fourth order cumulants:

$$\sum_{i=0}^{q}\mathbf{C}_{4,y}(2q-i,\tau_2-i,2q-i)h(i)=\alpha h(\tau_2-q) \tag{22}$$

with $\alpha=\left(\frac{\mathbf{C}_{4,y}(q,q,q)}{\mathbf{C}_{4,y}(q,q,0)}\right)^2\mathbf{C}_{4,y}(q,0,0)$.

Using the property of the cumulants $\mathbf{C}_{4,x}(\tau_1,\tau_2,\tau_3)=0$ if $\tau_1>q$, $\tau_2>q$ and $\tau_3>q$, the system of Eq. (22) can be written under the matrix form as follows:

$$\begin{bmatrix} \mathbf{C}_{4,y}(2q-1,q-1,2q-1) & .. & \mathbf{C}_{4,y}(q,0,q) \\ \mathbf{C}_{4,y}(2q-1,q,2q-1)-\alpha & .. & \mathbf{C}_{4,y}(q,1,q) \\ 0 & .. & . \\ . & . & . \\ . & . & . \\ . & . & . \\ 0 & .. & \mathbf{C}_{4,y}(q,q,q)-\alpha \end{bmatrix} \cdot \begin{bmatrix} h(1) \\ . \\ . \\ h(i) \\ . \\ . \\ h(q) \end{bmatrix} = \begin{bmatrix} \alpha-\mathbf{C}_{4,y}(2q,q,2q) \\ 0 \\ 0 \\ . \\ . \\ . \\ 0 \end{bmatrix} \tag{23}$$

The Eq.(23) can be written as follows:

$$A.\theta=d \tag{24}$$

with A is the matrix of size $(q+1,q)$ elements, the components zero in the matrix A correspond to cumulants having defined arguments outside of the definition region of the fourth order cumulants. θ is a column vector of size $(q,1)$ and d is a column vector of size $(q+1,1)$. The Least Square (LS) solution of the system of equation (24) is given by:

$$\theta=(A^TA)^{-1}A^Td \tag{25}$$

with A^T represents the transpose of A.

VI. Presentation of equalization model

In this section, we work on the blind equalization problem presented in the following figure [13]:

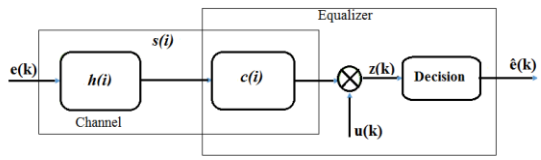

Fig. 2. The channel-equalizer system.

where:

- $\{e(k)\}$ is the channel input, $\{z(k)\}$ is the equalizer output, $\{u(k)\}$ is the **i.i.d** white noise moved to the output of the equalizer and $\{\hat{e}(k)\}$ is the estimated transmitted sequence.
- $\{h(i)\}_{i=0,\ldots,q}$ is the impulse response coefficients of the channel defined by: $h=[h(0)\ \ h(1)\ \ \ldots\ \ h(q)]^T$
- $\{c(i)\}_{i=0,\ldots,p}$ is the impulse response coefficients of the equalizer defined by: $c=[c(0)\ \ c(1)\ \ \ldots\ \ c(p)]^T$
- $\{s(i)\}_{i=0,\ldots,L}$ is the impulse response coefficients of the global system (channel+equalizer) defined by: $s=[s(0)\ \ s(1)\ \ \ldots\ \ s(L)]^T$ with $L=q+p$.

The objective of the equalizer is to find the coefficients $c(i)$ so we have

$$s(n)=\sum_{i=0}^{p}c(i)h(n-i)=\delta(n-D)=\begin{cases}1 & \textit{if} \quad n=D\\ 0 & \textit{if} \quad n\neq D\end{cases} \tag{26}$$

where D represents the transmission time delay, and the only non-zero coefficient with magnitude 1, i.e. the ideal equalization, for which the vector s is under the form:

$$s_d=[0\ \ \ldots\ \ 0\ \ 1\ \ 0\ \ \ldots\ \ 0]^T \tag{27}$$

The relation (26) can be written:

$$s=\mathbf{H}.c \tag{28}$$

where $\mathbf{H}$ is the matrix of $(L+1)$ lines and $(p+1)$ columns presented as follows:

$$\mathbf{H} = \begin{pmatrix} h(0) & 0 & \cdots & 0 & \cdots & 0 & 0 \\ h(1) & h(0) & \cdots & 0 & \cdots & \cdot & \cdot \\ \vdots & \vdots & \cdots & \vdots & \cdots & \cdot & \cdot \\ h(q) & \vdots & \cdots & h(0) & \cdots & \cdot & \cdot \\ 0 & h(q) & \cdots & h(1) & \cdots & 0 & \cdot \\ \cdot & 0 & \cdots & \vdots & \cdots & h(0) & 0 \\ \cdot & \cdot & \cdots & h(q) & \cdots & h(1) & h(0) \\ \cdot & \cdot & \cdots & 0 & \cdots & \vdots & h(1) \\ \cdot & \cdot & \cdots & \vdots & 0 & h(q) & \vdots \\ 0 & 0 & \cdots & 0 & \cdots & 0 & h(q) \end{pmatrix} \quad (29)$$

With the channel impulse response coefficients estimated by means of identification methods, we can use an algorithm based on minimizing the Mean Square Error (MSE) criteria defined as [14]:

$$J = E[|e(k) - z(k+D)|^2] \quad (30)$$

Using the assumptions that the transmission time delay D is known and the vector s has the form (27), we can derive the following formula to estimate the vector c [13]:

$$c = \left(\mathbf{H}^H.\mathbf{H} + \frac{\mu_{2,v}}{\mu_{2,e}} I\right)^{-1}.\mathbf{H}^H.s_d \quad (31)$$

where $\mathbf{H}^H$ represents transposed and conjugated $\mathbf{H}$, $\mu_{2,e}$ is the variance of the input sequence $\{e(k)\}$ and $\mu_{2,v}$ is the variance of the noise sequence $\{v(k)\}$.

Using the equation 4 with the output noise sequence $\{r(k)\}$, we have:

$$\mathbf{C}_{2,r}(\tau) = \mu_{2,e} \sum_{i=0}^{q} h^*(i)h(i+\tau) + \mu_{2,v}\delta(\tau) \quad (32)$$

The variance $\mu_{2,e}$ can be calculated by the following relation:

$$\mu_{2,e} = \frac{1}{q} \sum_{\tau=1}^{q} \frac{\mathbf{C}_{2,r}(\tau)}{\sum_{n=0}^{q} h^*(n)h(n+\tau)} \quad (33)$$

Exploiting the previous relation with $\tau = 0$ into Eq. (33), we can calculate the variance $\mu_{2,v}$ as:

$$\mu_{2,v} = \mathbf{C}_{2,r}(0) - \mu_{2,e} \sum_{i=0}^{q} |h(i)|^2 \quad (34)$$

VII. Simulation and discussion

We illustrate the performance of the proposed algorithm in the blind equalization of **GSM** channel modeled by an approximation of the following transfer function [15]:

$$\begin{aligned} H(z) = 1 &+ (0.9+0.6j)z^{-1} + (0.1250+0.8975j)z^{-2} \\ &+ (0.3261-0.7026j)z^{-3} + (-0.7162+0.2851j)z^{-4} \\ &+ (0.4422-0.6021j)z^{-5} \end{aligned} \quad (35)$$

The **PSK-8** constellations are used to generate the input sequence $\{e(k)\}$ of the channel, with a **SNR** ranging from $10dB$ to $40dB$. Fifty Monte-Carlo simulations were run, with the number of data points fixed at 4096 for each run. The number p of equalizer order and the time delay D were chosen to be equal to 31 and 15 respectively.

To evaluate the performance of the proposed algorithm, we compared it to the other algorithm [2] based on fourth order cumulants with the PSK constellations as input. Two criteria are used in this comparison: the representation of the impulse response of the global system and the inter-symbol interference ratio **(ISI)** defined by:

$$\mathbf{ISI}(dB) = 10\log\left(\frac{\sum_{j=0, j\neq d} |s(j)|^2}{|s(d)|^2}\right) \quad (36)$$

The table I summarizes the results of the simulation and comparison in different SNR. From this result, we notice that the algorithm ALG-Cum4 gives good results in term of ISI criteria compared to the other algorithm for all values of SNR. Figures 3 and 4 show the input and output equalizer constellations estimated by two algorithms ALG-Cum4 and Algorithm [2].

Data points N	**SNR**[dB]	**Methods**	**ISI** [dB]
4096	40	ALG-CUM4	−25.5485
		Algorithm [2]	−19.5559
	20	ALG-CUM4	−23.0902
		Algorithm [2]	−16.7816
	10	ALG-CUM4	−18.8165
		Algorithm [2]	−11.2285

TABLE I
INTERSYMBOL INTERFERENCE RATIO AS A FUNCTION OF **SNR**, FOR $N = 4096$.

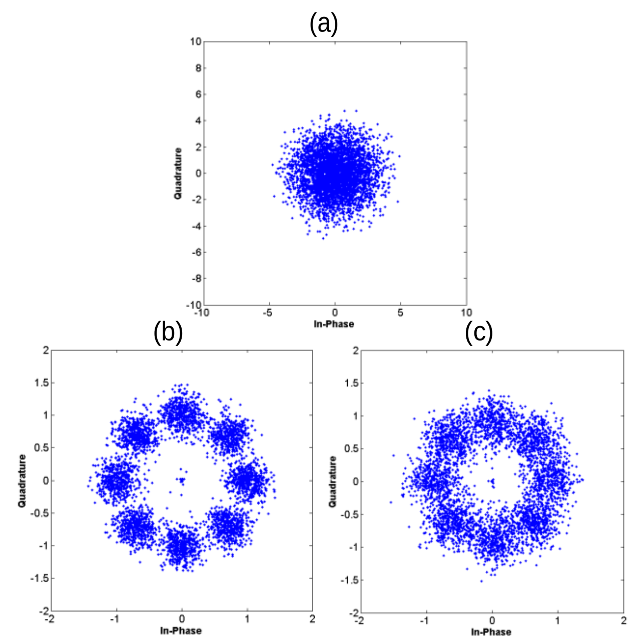

Fig. 3. Constellations of equalizer input and output for **SNR** = $10dB$ and $N = 4096$. (a) Equalizer input, (b) ALG-Cum4 algorithm, (c) Algorithm [2].

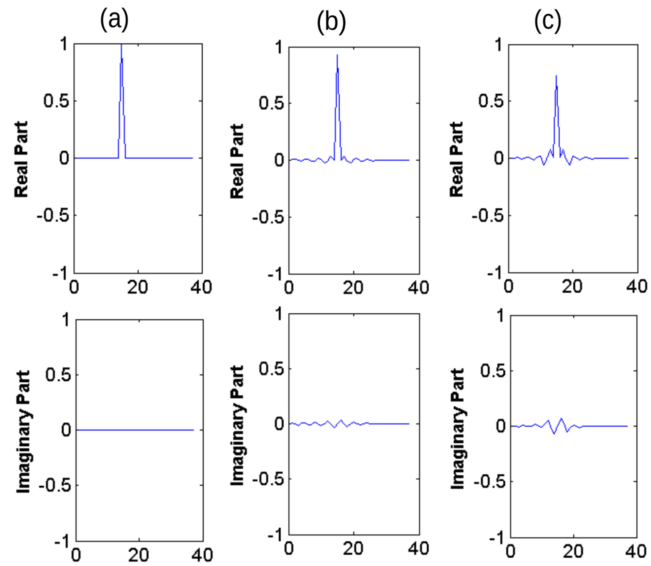

Fig. 4. Impulse response of global system for **SNR** = 10dB and N = 4096. (a) True, (b) ALG-Cum4 algorithm, (c) Algorithm [2].

From figures 3 and 4, we conclude that the equalization quality based on [2] algorithm deteriorates when we move towards a more noisy environment. The proposed algorithm ALG-Cum4 gives better results in that it is less sensitive to noise.

VIII. Conclusion

In this paper, we applied an algorithm based on fourth order cumulants on the equalization of communication channel in GSM. We compared it to another algorithm and ran a simulation using ISI criteria, signal-to-noise ratio (SNR) and impulse response of global system in imaginary and real parts. The results showed that the proposed algorithm was efficient with good constellations and presented the advantage of estimating the impulse response and the parameters of the equalization of communication channel GSM with accuracy.

References

[1] S. Safi and A. Zeroual. "Blind parametric identification of non-Gaussian FIR systems using higher order cumulants". International Journal of Systems Science, 35:15, 855-867, 2004.

[2] P. Comon, "MA identification using fourth order cumulants,". Signal Processing, vol. 26, no. 3, pp. 381-388, March 1992.

[3] J. Antari, R. Iqdour, and A. Zeroual. "Forecasting the wind speed process using higher order statistics and fuzzy systems ". Revue des Energies Renouvelables, 9 :237251, 2006.

[4] K. Abderrahim, R. Ben Abdennour, G. Favier, M. Ksouri and F. Msahli. "New results on FIR system identification using cumulants". APII-JESA. V 35 n 5 2001 p 601-622.

[5] J. M. Mendel."Tutorial on higher-order statistics (spectra) in signal processing and system theory: Theoretical results and some applications", Proc. of IEEE, vol. 79, pp. 278-305, 1991.

[6] C. L. Nikias, A. P. Petropulu." Higher-Order spectra analysis". Prentice Hall, Englewood Cliffs, New Jersey, 1993

[7] S. Safi, A. Zeroual. "MA system identification using higher order cumulants application to modelling solar radiation", Journal of Statistical Computation and Simulation, vol. 72, pp. 533548, 2002.

[8] S. Safi, A. Zeroual. "Blind identification in noisy environment of nonminimum phase finite impulse response (FIR) system using higher order statistics, Systems Analysis Modelling Simulation", vol. 43, pp. 671-681,2003.

[9] S. Safi, M. Frikel, A. Zeroual, and M. MSaad. "Higher Order Cumulants for Identification and Equalization of Multi-carrier Spreading Spectrum Systems ". Journal of telecommunications and information technology, January 2011.

[10] S. Safi, and A. Zeroual. " Blind Non-Minimum Phase Channel Identification Using 3^{rd} and 4^{th} Order Cumulants ". International journal of signal processing, V: 4 N: 1, 2007 ISSN 1304-4478.

[11] S. Safi, M. Frikel, M. MSaad and A. Zeroual. "Blind Impulse Response Identification of frequency Radio Channels: Application to Bran A Channel". International journal of signal processing, V: 4 N: 1, 2007 ISSN 1304-4478.

[12] D. R. Brillinger and M. Rosenblatt." Spectral Analysis of time Series, Chapter: Asymptotic Theory of k^{th}-order spectra ". New York, USA : Wiley, pages 153188, 1967.

[13] D. Dembl and G. Favier."A new FIR system identification method based on fourth-order cumulants: Application to blind equalization". Journal of the Franklin Institute January 1997.

[14] J. G. Proakis."Digital communications". McGraw-Hill International edition. 2^{nd} edition.

[15] K. Abderrahim."Identification de modles linaires l'aide des Statistiques d'Ordre Elev". Universit des Sciences, des Techniques et de Mdecine de Tunis: Thse de doctorat, Juillet 2000.